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Gordon Kissam

Jan. 1912

F. M. S.

1. Arithmetic. — Textbooks (Ele[illegible] 19[illegible]

ELEMENTS
OF
ARITHMETIC
MENTAL AND WRITTEN

NEW SERIES

BY THE
BROTHERS OF THE CHRISTIAN SCHOOLS

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PREFACE.

THE present volume is the first of a series of Arithmetics prepared in accordance with the methods in use in the classes of the Brothers of the Christian Schools. Those methods are the result of the united experience flowing from two centuries of teaching. Their chief merit is this: that they dispense with the old way of simply getting the rules by rote and then applying them; in the stead, they speak to the child's intelligence, and lead him up gradually to the understanding of the principles upon which the rules are based. Knowing both the rule and the reason for it, the child no longer labors in the dark, and the teaching of Arithmetic ceases to be mere machine-work. He may afterwards forget the rule; but having learned to reason the problem out independently of a set form of words, he finds no difficulty in reaching the solution of any ordinary case to which he may apply himself.

Indeed, there is no other efficient method of teaching Arithmetic than this of appealing to the child's intelligence. The correct solution of every problem is an exercise of judgment. It is therefore a good means of mental discipline. Mere memory-work will not produce the same result; it will rather cramp and impede the other faculties. It is not the method of nature. From the moment the child has learned to speak he seeks to know the why and wherefore of things, and ceases to ask questions only when he becomes discouraged by the unsatisfactory answers of inconsiderate parents or incompetent teachers.

All practical life is rooted in number and calculation. Business is transacted upon a basis of counting. Hence the importance of teaching this subject according to approved and practical methods. For this reason, our SCHOOL'S GOVERNMENT lays special stress upon accurate rather than rapid calculation. It says: "*In teaching Arithmetic it is of less importance to teach the pupil to calculate rapidly than to do so accurately.*" It furthermore insists, as a means of acquiring habits of accurate thinking, that precise language be used in all explanations and calculations: "*The pupils should be taught to make use of the correct terms, and never in their work to employ a useless expression.*" And again it says: "*We should insist on the pupils being very exact in the recitation of definitions and principles.*" Finally, it counsels the exercise of the pupil's judgment in words to which nothing can be added: "*After having read the problem to be solved, and written on the blackboard the conditions it contains, the pupils should be required to give an ac-*

count, first, of what is asked of them, or what they have to find out; second, of what they know in reference to it; third, of the operations required by the nature of the problem."

The following pages will be found to embrace these principles in every respect. The language is adapted to the young minds for which it is intended. The explanations aim at being simple. The solutions given are strictly in accordance with the definitions and principles laid down. But the teacher need not confine himself to the forms here given. After the pupil understands these, he may be shown other methods with profit.

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ELEMENTS OF ARITHMETIC.

INTRODUCTORY DEFINITIONS.

1. ***Arithmetic*** is the *science* of *numbers,* and also the *art* of *computation.*

2. ***Number*** is the *result* of the *comparison* of a *quantity* with *unity.*

3. ***Quantity*** is anything that can be increased, diminished, or measured; as, the *length* of a road, the *surface* of a body, the *weight* of an article.

4. A ***Unit*** is a quantity with which we compare others of the same kind.

5. The comparison of quantity with unity produces three kinds of numbers: *Integers, Fractions,* and *Mixed Numbers.*

6. An ***Integer*** is a number which contains its unit an exact number of times: as, 12, 15; 6 boys, 4 apples.

7. A ***Fraction*** is a number which is less than a unit; as, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{7}{8}$.

8. A ***Mixed Number*** consists of an integer and a fraction. Thus, $1\frac{1}{2}$, $2\frac{1}{3}$, $4\frac{1}{4}$.

9. According to the nature of their unit, numbers are divided into two classes; viz., *Abstract* and *Concrete.*

10. An ***Abstract Number*** is a number the nature of whose unit is not determined; as, 16, 425, 7840.

11. A ***Concrete Number*** is a number the nature of whose unit is determined; as, 16 men, 425 days, 7840 dollars.

TERMS EMPLOYED

1. Arithmetical Operations are the divers changes to which numbers are subject.

There are four fundamental operations: *Addition*, *Subtraction*, *Multiplication*, and *Division*.

2. The ***Proof*** of an operation is a second operation made in order to verify the correctness of the first. The proof does not give an absolute certainty, but only a great probability, that the operation has been correctly performed.

3. A ***Problem*** is a question to be solved.

4. The ***Resolution*** of a problem consists in finding the unknown quantities by means of the known.

The resolution of a problem comprises both the *solution* and the *calculation*, or *operation*.

5. The ***Solution*** is the series of operations necessary to arrive at the required result.

6. The ***Calculation*** is the performance of the operations indicated by the solution.

NOTE.—The calculation refers exclusively to abstract numbers or those considered as such.

7. A ***Principle*** is a fundamental truth from which others are derived.

8. A ***Rule*** is a statement of the method of obtaining a desired result.

NUMERATION AND NOTATION.

12. ***Numeration*** is the method of reading numbers expressed by characters.

13. ***Notation*** is the method of writing numbers.

14. Numbers may be represented as follows:

I. By *words;* as, one, two, three.

II. By *figures,* called the *Arabic Method;* as, 1, 2, 3.

III. By *letters,* called the *Roman Method;* as, I, V, X, C.

15. In the ***Arabic Method,*** numbers are expressed by the following ten

Figures:	1,	2,	3,	4,	5,	6,	7,	8,	9,	0.
Names:	One,	Two,	Three,	Four,	Five,	Six,	Seven,	Eight,	Nine,	Naught.

16. The first nine figures are called *significant* because they represent a value. But the tenth, by itself, represents nothing. It is only an auxiliary figure; its office being to hold the place of any order whatever, when there are no units of that order in the number.

17. Each of the first nine numbers expresses simple units, or units of the *first order.*

18. The number which follows the ninth is called *ten.* It is represented by writing the figure 1 with a naught after it; thus, 10.

19. *Ten* is the unit of the *second* order, and is equal to ten units of the *first* order.

20. We count by tens as we count by simple units, saying: *one ten, two tens, three tens, nine tens.* But custom has replaced these words by the following:

Twenty,	20.	Sixty,	60.
Thirty,	30.	Seventy,	70.
Forty,	40.	Eighty,	80.
Fifty,	50.	Ninety,	90.

NOTE.—The "ty" in these words signifies ten.

21. The names of the numbers included between two consecutive tens, are formed by joining to the name of the first of these tens, the name of each of the first nine numbers, saying:

Twenty-one,	21.	Thirty-one, etc.	31, etc.
Twenty-two,	22.	Forty-one, etc.	41, etc.
Twenty-three,	23.	Fifty-one, etc.	51, etc.
Twenty-four,	24.	Sixty-one, etc.	61, etc.
Twenty-five,	25.	Seventy-one, etc.	71, etc.
Twenty-six,	26.	Eighty-one, etc.	81, etc.
Etc.	Etc.	Ninety-one, etc.	91, etc.

The highest number expressed by two figures, being ninety-nine, 99.

22. But instead of saying ten and one, ten and two, ten and three, ... ten and nine, custom has adopted the expressions:

Eleven,	11.	Fifteen,	15.
Twelve,	12.	Sixteen,	16.
Thirteen,	13.	Seventeen,	17.
Fourteen,	14.	Eighteen,	18.
	Nineteen,	19.	

NOTE.—The "teen" in the words thirteen, etc., to nineteen, means *ten.* So that, strictly speaking, thirteen means three and ten, fourteen, four and ten, etc.

23. The number which follows ninety-nine (99) is called *hundred.* It is represented by writing 1 with two naughts after it; thus, 100.

24. *One hundred* is the unit of the *third order,* and is equal to ten units of the *second order.*

25. We count by hundreds as we count by units, saying:

One hundred,	100.	Five hundred,	500.
Two hundred,	200.	Six hundred,	600.
Three hundred,	300.	Seven hundred,	700.
Four hundred.	400.	Eight hundred,	800.
		Nine hundred,	900.

26. The names of the numbers included between two consecutive hundreds, are formed by joining, successively, to the name of the first of these hundreds, the names of all the numbers less than one hundred, thus:

One hundred one,	101.
One hundred two,	102.
One hundred three,	103.
Eight hundred eighty-six,	886.
Nine hundred ninety-eight.	998.

Nine hundred ninety-nine (999) is the highest number that can be expressed by three figures.

27. The group comprising the first three orders of units, viz., *units, tens,* and *hundreds,* constitutes the *first period,* that of simple units.

28. The number which follows 999 is called *thousand,* and is represented by writing the figure 1 with three naughts after it; thus, 1000.

29. *Thousand* is the unit of the *second period.* The period of *thousands*, like that of simple units, comprises units, tens, and hundreds. The units of thousands, tens of thousands, and hundreds of thousands, constitute the fourth, fifth, and sixth orders of units.

The units of thousands are:

One thousand, two thousand, ... nine thousand.
1000, 2000, 9000.

The tens of thousands are:

Ten thousand, twenty thousand, ... ninety thousand.
10000, 20000, 90000.

The hundreds of thousands are:

One hundred thousand, two hundred thousand,
100000, 200000,
..............nine hundred thousand.
900000.

30. The names of the numbers between two consecutive *orders* of thousands, are formed by joining, successively, to the name of the first of these orders, the names of all the numbers less than this order. In this manner we reach the number 999999.

31. Continuing in the same manner, we form the next higher periods, *Millions, Billions, Trillions, etc.*

Million is the unit of the *third* period; billion, the unit of the *fourth* period; and trillion, the unit of the *fifth* period.

These three periods, like *units* and *thousands,* comprise, each, three orders; viz., the order of units, that of tens, and that of hundreds.

32. The names of the various numbers included between their several orders are formed in the same way as those included between the several orders of thousands.

33. Every significant figure has two values. One is called its *simple,* or *absolute* value; and the other, its *local,* or *relative* value.

The *Simple Value* of a figure is that given to it by its form.

The *Local Value* is that which it receives from the place that it occupies in the number.

Thus in the number 4306, the simple value of the first figure to the left is 4; and its local value is 4 units of thousands.

34. For convenience in reading and writing numbers, the figures are divided into *periods,* each of which comprises three places. The *first three* places constitute the *first,* or *units* period; the *second three* places constitute the *second,* or *thousands* period; etc.

35. This division of the periods will be easily understood by a careful examination of the following

NUMERATION TABLE.

Hundreds of Tens of Units of **Trillions,**	Hundreds of Tens of Units of **Billions,**	Hundreds of Tens of Units of **Millions,**	Hundreds of Tens of Units of **Thousands,**	Hundreds of Tens of Units of **Units.**
840	625	074	503	040
89	031	209	654	061
30	005	800	237	500
201	892	604	083	275
2	000	596	000	082
987	015	003	106	000
400	000	000	500	005

36. If it be required to read or write numbers above trillions, the following is the order of some of the next higher periods: *Quadrillions, Quintillions, Sextillions, Septillions, Octillions, etc.*

EXERCISES IN NUMERATION.

1. What number is expressed by 75346821 ?

SOLUTION.—Separating these figures into periods, we have **75**, 346, 821. The third period is 75 *millions;* the second is **346** *thousands;* and the first is 821 *units;* hence, the number is 75 millions, 346 thousands, 821.

NOTE.—If an order, or even an entire period, be wanting, we do not mention it. We also omit to name the last or units period, because it is understood.

Read the following numbers:

1.	*2.*	*3.*	*4.*	*5.*
75	972	1008	60001	704825
63	840	5000	73182	659037
37	569	6300	39502	954308
25	708	2501	18007	100716
12	411	15784	40905	536900
50	4934	29092	56000	213472
225	6527	56311	171360	360005
121	4025	12102	562984	800001
309	7690	20976	630192	780602
406	2416	10083	935871	170112
721	3083	84960	214650	756640
582	6135	37251	836422	151312
391	1237	45162	178245	702514
420	6030	24310	790031	953716
569	1191	37521	110725	803556
230	1688	89111	333470	678520

6.

5006351
14500021
7040306
907001
8200000
60500005
9008600
20036409
460072908
111000111
305009705
99092666
200067425
180990040

7.

4072634
82791640
7006
984270
400200
1807
62876000
900040
900000800
3742680002
8632073009
862794846704
2872819642
3400641111

8.

3563400024
7462007302
82367400210
53024046070
86920000030
17629080406
294635112211
909009008007
842780062004
1303000170410
3278642197416
14000075001004
167008634216786
3462184390075819

EXERCISES IN NOTATION.

1. Express in figures the number *two thousand four hundred three.*

SOLUTION.—This number consists of two periods, *thousands* and *units.* In the *thousands* period there is but one order, which is 2 units of thousands. We, therefore, write 2 thousands in the fourth place, 4 hundreds in the third place, a naught (0) in the second place, there being no *tens,* and 3 units in the first place. Hence, these figures, 2403, are the proper expression for the given number.

Express the following numbers in figures:

2. Thirteen. Eleven. Seventeen.
3. Nineteen. Thirty-four. Eighty-two.
4. Fifty-five. Ninety-four. Forty-three.
5. Three hundred sixty. Five hundred eighty-two.
6. Nine hundred twelve. Three hundred fourteen.
7. Seven hundred nineteen. Nine hundred three.

8. Nine thousand, seven hundred eight.

9. Eighteen thousand, seven hundred.

10. Fifty thousand, nine hundred one.

11. Seventy thousand ten.

12. Two hundred ninety-one thousand, seven hundred fifty-three.

13. Six hundred seventy-three thousand, twenty-four.

14. One million, four hundred thirty-two thousand, three hundred sixteen.

15. Fourteen million, thirty-five thousand, one hundred ninety-four.

16. One million, three.

17. Seven million, three hundred thousand, ninety-four.

18. Forty million, four thousand, seven hundred.

19. Six hundred three million, fifteen thousand, sixty-one.

20. Fifteen billion, seventy-one million, six thousand, four hundred.

21. Three hundred thousand, five hundred eighty-two.

22. Two hundred million, fourteen thousand, one hundred.

23. Eight hundred thirty billion, twenty thousand, twenty-two.

24. Five million, two hundred six thousand, nineteen.

25. Nine hundred billion, sixteen million, eight thousand.

26. One hundred nine million, four hundred twenty thousand.

27. Five hundred twenty-one million, three thousand, ten.

28. One hundred two billion, two hundred seventy thousand, ten.

29. Twenty-seven billion, fifty million, five hundred ninety-one.

30. Three hundred million, seventy thousand, nine hundred.

31. Three trillion, one hundred twenty billion, two million, five thousand, one.

ROMAN NOTATION.

37. In the *Roman Method* of Notation, numbers are expressed by the following seven letters of the Roman Alphabet:

Letters.	I,	V,	X,	L,	C,	D,	M.
Values.	1,	5,	10,	50,	100,	500,	1000.

PRINCIPLES.—*I. The value of the letter is repeated as often as the letter itself is repeated; as, III expresses the number three; XX, expresses twenty.*

II. A letter placed to the right of one of greater value, adds its own to that of the other; as, XV represents fifteen; VII, seven.

III. The value of a letter placed to the left of one of greater value, must be subtracted from that of the other; as, IV expresses four; IX, nine.

IV. The value of a letter or a combination of letters, is increased a thousand-fold by placing a dash over it. Thus, $\overline{X}$, $\overline{LX}$, *denote, respectively, ten thousand, and sixty thousand.*

NOTES.—1. If a letter that denotes a less number be placed between two that denote greater numbers, it diminishes the latter, but does not affect the former. Thus, in the combination LIX, the value of I must be taken from that of X. Hence, the number expressed is fifty-nine (59).

2. It must be observed also that no letter is written four times in succession.

3. There are only six combinations in which a character is placed before one of greater value. Hence, the numbers, 4 (IV), 9 (IX), 40 (XL), 90 (XC), 400 (CD), and 900 (CM), being represented by peculiar double characters, should not be expressed by any other combinations.

38. The application of these principles is shown in the following

TABLE.

I	One	XIX	19
II	Two	XX	20
III	Three	XXX	30
IV	Four	XL	40
V	Five	L	50
VI	Six	LX	60
VII	Seven	LXX	70
VIII	Eight	LXXX	80
IX	Nine	XC	90
X	Ten	C	100
XI	Eleven	CC	200
XII	Twelve	CD	400
XIII	Thirteen	D	500
XIV	Fourteen	DC	600
XV	Fifteen	CM	900
XVI	Sixteen	M	1000
XVII	Seventeen	MM	2000
XVIII	Eighteen	MDCCCLXXXIII	1883

Note.—This system of notation is named after the Romans by whom it was invented and used. It is now principally confined to numbering chapters, sections of books, public documents, etc.

EXERCISES.

Read the following numbers and express them in figures:

1.	*2.*	*3.*	*4.*
IV	XXII	LXXXIII	CMLX
XV	XXXII	XIV	DCVIII
XLIV	XVI	LXXXVIII	CXIV
LXXV	LV	LXX	DCLIII
XXVIII	LI	LIX	DCCXLIII
XXXIX	LXII	XCIII	$\overline{V}$DCCXI
XI	XCI	XXIII	$\overline{IV}$XC
XLIX	LXXVIII	XCVII	$\overline{D}$XLIX

Write the following numbers by the *Roman Method:*

1.	*2.*	*3.*	*4.*	*5.*	*6.*	*7.*
17	24	34	30	830	462	262
45	18	46	67	561	689	109
63	57	12	70	326	984	476
71	64	89	82	279	533	324
25	38	77	96	195	372	712
36	27	98	60	914	607	416
52	13	41	29	182	309	967

8.	*9.*	*10.*	*11.*	*12.*
1876	1776	2132	2358	6908
1882	1860	3004	3422	12674
1512	1783	4040	4004	11492
1492	2579	7632	8743	21800
1600	1900	2250	4100	15000
1888	1777	3045	3009	12650
1865	1420	2804	7180	10800

ADDITION.

39. ***Addition*** is the process of finding the sum of two or more numbers of the same kind.

40. The ***Sum*** of several numbers is a number which contains as many units as the numbers added.

41. The ***Sign of Addition,*** +, is called *plus*. This sign denotes that the numbers between which it is placed are to be added. Thus, the expression, 6 + 3, shows that 3 is to be added to 6.

OPERATION OF ADDITION.

1. Let it be required to find the sum of 475, 854, and 696.

OPERATION.

$$\begin{array}{r} 475 \\ 854 \\ 696 \\ \hline 2025 \end{array}$$

SOLUTION.—Having written the numbers so that units of the same order stand in the same column, we begin at the right and add each column separately. The sum of 6 units and 4 units is 10 units; and 10 units and 5 units are 15 units, which are equal to 1 ten and 5 units. We write the 5 units under the column of units, and carry the 1 ten to the column of tens. We next add the column of tens, beginning with the 1 ten which we carried from units' column. The sum of 1 ten and 9 tens is 10 tens, and 5 tens are 15 tens, and 7 tens are 22 tens; that is, 2 hundreds and 2 tens. Writing the 2 tens in the column of tens, we carry the 2 hundreds to the column of hundreds. The sum of the hundreds thus increased, 2 + 6 + 8 + 4, is 20 hundreds; that is, 2 thousands and 0 hundreds. As this is the last column, we set down the entire sum. The number, 2025, is, therefore, the required sum, because it is the sum of the units, tens, and hundreds of the given numbers.

Proof.—Perform the operation by commencing at the top of each column and adding downward. If the same result be obtained, it is probable that the work is correct.

ILLUSTRATIONS.

2.	*3.*	*4.*	*5.*	*6.*
412	49	418	4734	4500
343	716	36	8062	982
917	503	672	191	3799
1672	1268	1126	12987	9281

7.	*8.*	*9.*
542 feet.	260 days.	278 pounds.
717 “	428 “	402 “
203 “	600 “	736 “
971 “	26 “	877 “
2433 feet.	1314 days.	2293 pounds.

Note.—The operation of adding a column of figures should be abbreviated by simply naming the result of each step. Thus, in example 7, the pupil should say 1, 4, 11, 13 (1st column); 1, 8, 9, 13 (2d column); and 1, 10, 12, 19, 24 (3d column).

WRITTEN EXERCISES.

10.	*11.*	*12.*	*13.*	*14.*
3141	9316	3345	6205	1073
9071	8200	8801	3903	2208
1625	7510	7865	6675	7762
9301	8715	4322	1885	1839
1870	1930	3418	1907	3033
2415	6740	1879	3314	4111
1980	3972	2860	2970	6395
2634	4060	1790	6327	2136

15.	*16.*	*17.*	*18.*
57082	54133	16098	231872
13221	12235	44120	123987
10939	63798	39862	203524
24532	23787	55032	161724
19980	70390	73852	230543
20018	23901	33487	130107

19.	*20.*	*21.*
4 292	874	704 854 927
468 356	68 422	305 476 507
9 746 940	8 792 624	999 767 689
903 806 706	349 508 628	524 783 876
504 784 949	45 302 011	763 525 202
25 376 458	948 306 405	889 731 633

22.	*23.*	*24.*
67 504	6 245 754	42 085
536 789	272 879 182	45 690
547 645 978	94 807 503	649 743
78 456 708	786 546 597	8 437 348
897 604 847	70 964	497 063 547
89 478 600	70 368 483	594 627 795
976 343 269	526 934	99

25. Four hundred fifty thousand, two hundred twenty; two hundred thirty-one thousand, eighty-six; one million, two hundred sixty-two thousand, seven hundred one; four hundred one thousand, six hundred eighteen.

26. Nine thousand; seven hundred thirty-eight; five thousand; seven hundred eighty-nine; six thousand, three hundred four; two hundred thirty-eight thousand.

27. Three thousand, one hundred twenty; two hundred six; five hundred sixteen thousand, three hundred forty-seven; nine hundred thousand, three hundred forty; and five hundred twenty-six.

28. Sixty thousand, seven hundred eighty-five; thirteen thousand, four hundred and seventy-eight; ninety-six thousand, five hundred seventy-one; five hundred eight; twelve thousand, six; six hundred eighty-two; and one thousand, seventy-three.

29. 99 million, 127 thousand, 806; 73 million, 156 thousand, 4 hundred 25; 130 million, 17 thousand, 96; 72 million, 45 thousand, 14.

30. 67 million, 849 thousand, 341; 110 thousand, 389; 63 million, 448 thousand, 720; and seven hundred eight.

31. 5 thousand, 163; 12 thousand, 2; 100 thousand, 9; 1 billion, 100 million, 110 thousand, 11; and nine hundred seven.

32. 90 billion, 90; 10 thousand, 6 hundred; and 200 thousand, 780.

33. 87 million, 63 thousand, 11; 14 thousand, 60; 3 million, 3 thousand, 3; 72 thousand, 601; 84 million, 15 thousand, 107; and 162 million, 7 thousand, 23.

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42. The ***Sign,*** $, written before a number signifies *dollars.* Thus, the expression $120 is read *one hundred twenty dollars.*

43. *Dollars* and *cents* may be written together, the *cents* being separated from the *dollars* by a point. Thus the expression $25.35 is read 25 *dollars* and 35 *cents.*

When the number of cents is less than 10, a naught must be placed between the point and the figure representing the number of cents. Thus, 6 dollars and 7 cents are written $6.07.

Cents, when written alone, may be expressed either by placing $0. before them, or by simply using the word cents. Thus, *twenty-seven cents* are written $0.27, or 27 cents.

WRITTEN EXERCISES.

Express by proper signs and figures the following:

1. Seven dollars and twenty-four cents. Five dollars.
2. Sixteen dollars and forty cents. Thirty-four cents.
3. Eighty-seven cents. Four dollars and nine cents.
4. Seventy-five cents. Seven dollars and six cents.
5. Eleven dollars and eleven cents. Nine dollars.
6. Fifty dollars and twenty-five cents. Forty cents.
7. Nineteen dollars and three cents. Sixty dollars.
8. Fifty cents. Eighty-five cents. Eighteen cents.
9. Three dollars and fifty cents. Ninety-two cents.
10. Sixty-two dollars and nine cents. Thirty cents.

44. In writing dollars and cents for the purpose of adding them, the separating points must stand in the same column.

1. What is the sum of $10.27, $123.06, $206.90, $3.10, and $0.46?

OPERATION.

```
$10.27
123.06
206.90
  3.10
  0.46
-------
$343.79
```

SOLUTION.—Having arranged the numbers according to **Art. 44,** we add them by the principles of simple addition. The separating point is placed in the result immediately under those of the given numbers.

WRITTEN EXERCISES.

In like manner add the following:

2.	*3.*	*4.*	*5.*
$31.05	$217.75	$78.50	$981.34
54.62	83.16	151.63	89.65
125.84	55.32	96.18	326.10
370.62	135.67	12.03	4823.63

6.	*7.*	*8.*	*9.*
$65.14	$75.15	$2245.76	$30000.00
91.73	207.45	791.18	4706.50
182.64	1241.16	33.87	373.33
79.30	79.67	6.75	876.45
20.37	34.81	650.37	1950.55

MENTAL EXERCISES.

1. A grocer sold 67 quarts of strawberries one day and 28 quarts another; how many quarts did he sell in both days?

2. A carpenter earned 54 dollars one month and 76 the next; how much did he earn in the two months?

3. A man bought a horse for $190, and a wagon for $150; what was the cost of both?

4. Eliza is 30 years older than Mary, and Mary is 20 years old; how old is Eliza?

5. Some muslin cost 30 cents, and some silk 400 cents; what was the cost of both?

6. Edward lost 40 cents, and has 30 cents left; how much had he at first?

7. Thomas has studied 60 lessons this month, which is 20 less than the number Francis has studied; how many lessons has Francis studied?

8. A drover purchased 125 sheep at one time and 113 at another; how many did he buy on both occasions?

9. A boy has 14 cents in his vest pocket and 23 in his pants pocket; how many cents has he in both pockets?

10. My cousin gave me 76 marbles, and I had 140 before; how many marbles have I now?

11. William has 36 photographs in one album and 48 in another; how many photographs in both albums?

12. A farmer sold 62 barrels of apples and has 103 barrels remaining; how many barrels had he at first?

13. A boy sold a book for 28 cents less than it cost. If he received 47 cents, what was the buying price?

14. There are 39 roses on one bush and 26 on another; how many roses on both bushes?

15. While repairing a factory 157 men were engaged on the exterior and 33 on the interior; how many men were employed?

16. John's school is 134 rods north of his residence, while his brother Martin's is 63 rods south; how far are the two schools apart?

WRITTEN EXERCISES.

1. A man bought a horse and carriage for $520, a cow for $65, and a wagon for $90. How much did he pay for all?

2. I own a farm worth $56000, and a house worth $30800; how much are both worth?

3. Three men invested in cotton; A invested $7560; B invested $9200; and C invested $12300. How much did they all invest?

4. An army consists of three grand divisions; the first contains 60400 men; the second 47400 men; and the third 85000 men. How many men in the army?

5. A lady paid $85 for a carpet, $60 for some chairs, $25 for a bureau, and $20.50 for a table. How much did she pay for all?

6. If there are 163 sheep in each of 5 pastures, how many sheep in all?

7. Find the sum of all the numbers from 136 to 146.

8. After paying $375.50 to the grocer, $460 to the butcher, and $89.50 to the tailor, Mr. Johnson has $784.90 left. How much had he before he paid his debts?

9. A merchant pays his foreman $1200 a year, three salesmen $640 each, a porter $500, and an errand boy $200. How much does he pay per year in salaries?

10. A grocer sold 10 chickens for $4.98; 6 ducks for $2.45; and 7 turkeys for $9.36. How many fowls did he sell, and what were they worth?

11. A man bought a set of furniture for the parlor, which cost $483.75; one for the sitting-room, costing $265.50; a chamber set for $180.25; and a set for the dining-room, costing $225.90. What was the cost of the four sets? *Ans.* $1155.40.

12. A gentleman donated the following sums for charitable purposes: $4840.65; $7219.80; $975.50; $3265.20; $1783.85. How much did he donate in all?

13. The property of Mr. McLaughlin was valued as follows: land, $15800; house, $3475; barn, $950; horse, $225; carriage, $175; farm implements, $1675; and money in bank, $4900. What was the entire value of his property? *Ans.* $27200.

14. A gentleman owes the following: to Mr. Smith, $67.50; to Mr. Murphy, $122.75; to Mr. Jones, $89.25; to his butcher, $185.62; to his baker, $75.87; and to his grocer, $220.55. What is the amount of his debts?

Add by lines, and by columns, the following:

LEDGER COLUMNS.

	A.	B.	C.	D.	E.	Total.
	8999 6					
1st...	$584.25	$1060.15	$1200.40	$980.00	$103.04	*** **
2d....	79.48	320.80	915.16	90.50	27.01	*** **
3d....	104.28	75.00	812.08	71.55	4005.80	*** **
4th...	2600.00	84.30	2510.24	183.80	981.10	*** **
5th...	403.50	2100.00	630.12	320.42	49.19	*** **
6th...	694.20	64.50	100.25	201.60	800.66	*** **
7th...	1630.75	790.25	4300.95	500.40	313.40	*** **
8th...	15.80	34.00	650.10	63.24	1500.73	*** **
9th...	210.00	815.70	104.50	1000.00	2014.47	*** **
10th...	521.00	431.65	813.60	720.00	613.09	*** **
11th...	69.50	92.80	25.00	50.13	53.84	*** **
12th...	141.12	200.00	161.00	666.78	299.70	*** **
13th...	3260.00	99.00	35.16	1428.00	14.16	*** **
14th...	914.63	71.15	72.00	1126.30	313.91	*** **
15th...	25.00	370.83	912.50	47.21	808.05	*** **
16th...	711.00	1506.90	85.48	11.60	3020.80	*** **
17th...	39.48	400.50	600.12	783.13	706.09	*** **
18th...	1050.65	510.09	566.86	52.51	98.89	*** **
19th...	705.00	1085.00	1022.25	412.17	1206.25	*** **
20th...	874.50	37.48	203.50	97.00	591.75	*** **
21st...	1020.15	4.65	16.73	9000.00	610.00	*** **
22d....	341.00	19.37	9.87	400.00	50.50	*** **
23d....	97.50	114.20	35.48	96.50	100.25	*** **
24th...	2014.25	2917.62	300.16	39.85	312.75	*** **
25th...	120.90	4.96	5000.50	114.76	8.41	*** **
26th...	251.12	12.88	700.35	2.15	800.70	*** **
27th...	305.85	3.75	36.82	83.35	2619.38	*** **
Total..	**** **	**** **	**** **	**** **	**** **	**** **

NOTE.—In adding long columns, it will be found convenient to write in small figures, at the top of each column, the number carried from the preceding column, as shown in No. A.

SUBTRACTION.

45. ***Subtraction*** is the process of finding the difference between two numbers of the same kind.

46. The ***Minuend*** is the greater number.

47. The ***Subtrahend*** is the smaller number.

48. The ***Difference*** is the number obtained by subtracting.

49. The ***Sign of Subtraction,*** **–,** is called ***minus.*** It denotes that the number immediately following it is to be subtracted from the number preceding it; as, in the expression, 8 – 5, the sign minus signifies that 5 is to be subtracted from 8.

OPERATION OF SUBTRACTION.

1. Let it be required to find the difference between 837 and 564.

OPERATION.

Minuend,	837
Subtrahend,	564
Difference,	273

SOLUTION.—We write the subtrahend under the minuend so that units of the same order shall stand in the same column. Beginning at the right, we see that 4 units from 7 units leave 3 units, which we write in the line below. Since 6 tens cannot be taken from 3 tens, we borrow one hundred, or ten tens, from the 8 hundreds, and add it to the 3 tens, making 13 tens. Now, 6 tens from 13 tens leave 7 tens, which we write under the tens. To compensate for the 10 tens, or 1 hundred added to the minuend, we diminish the 8 hundreds by one hundred. Then, 5 hundreds taken from 7 hundreds, leave 2 hundreds, which is written under hundreds.

The number 273 is, therefore, the difference between the two given numbers; because it is the sum of the several remainders, obtained by subtracting the parts of the subtrahend from the corresponding parts of the minuend.

Proof.—Add the difference to the less, and if the work be correct, the result will give the greater number.

ILLUSTRATIONS.

	2.	*3.*		*4.*
From	5736	587673	yards.	$5820.54
take	3428	93736	"	2074.18
Difference,	2308	493937	"	3746.36
Proof,	5736	587673	"	$5820.54

WRITTEN EXERCISES.

5. 634 — 231.
6. 635 — 308.
7. 273 — 156.
8. 420 — 116.
9. 481 — 375.
10. 9655 — 4479.
11. 3542 — 2334.
12. 2736 — 1786.
13. 4087 — 3754.
14. 6321 — 5213.
15. 71583 — 53866.
16. 38293 — 16532.
17. 15665861 — 4329095
18. 87593647 — 7825636
19. 59020000 — 1009284.
20. 32670941 — 8732804.
21. 79302274 — 9304506.
22. 10627304 — 10416208.
23. 73462894 — 62187326.
24. 598354987 — 98708905.
25. 507205304 — 408609927.
26. 258245071 — 47347897.
27. 866477955 — 499393676.
28. 487500654 — 90579864.
29. 408253695 — 406211203.
30. 753402500 — 96677657.
31. 957874009 — 894377258.
32. 100640001 — 30786203.
33. 320600021 — 253060522.
34. 1290478132 — 849534119
35. 6784000091 — 590090099
36. 1100122301 — 941098877

UNITED STATES CURRENCY.

50. To subtract *dollars* and *cents* write them as in addition, so that the separating points may fall in the same column.

1. From $26.03 subtract $17.67.

OPERATION.

$26.03, Minuend.
17.67, Subtrahend.
$8.36, Difference.

SOLUTION.—Place the less number under the greater, taking care to have the points in the same column. Then proceed as in ordinary subtraction.

ILLUSTRATIONS.

	2.	*3.*	*4.*	*5.*
From	$39.62	$186.25	$170.00	$2084.62
take	14.37	49.75	37.33	1950.27
	$25.25	$136.50	$132.67	$134.35
	6.	*7.*	*8.*	*9.*
	$2500.00	$360.01	$8100.75	$10760.00
	1750.20	73.09	998.63	8700.75

WRITTEN EXERCISES.

10. Purchased a farm for $16787.99 and sold it for $18000; what was my gain? *Ans.* $1212.01.

11. A tailor bought cloth to the amount of $63.25 and afterward sold it for $59.16; what was his loss?

Ans. $4.09.

12. A and B began business with a capital of $16000. If A put in $9713.73, what was B's share of the capital?

Ans. $6286.27.

13. A gentleman having $3800.25 in bank, drew out $468.71; how much has he remaining in bank?

Ans. $3331.54.

MENTAL EXERCISES.

1. A boy had 75 cents and spent 37 cents; how much had he left?

2. Joseph bought 30 peaches, gave 8 to John, and eat 10; how many had he left?

3. A lady bought some tape for 25 cents, some needles for 15 cents, and some thread for 20 cents; how much less than a dollar did all cost?

4. The sum of two numbers is 32; one of them is 18; what is the other?

5. John had $35, and earned $17 more. If he spend $22, how much will he have left?

6. The difference of two numbers is 19, and the greater is 56; what is the less?

7. A young man completed his college course at the age of 21, having taken 13 years for that purpose; how old was he when he entered college?

8. I sold a cow for $60, which was a loss of $12; what did I pay for it?

9. A boy lost 50 marbles at play, then bought 20, and found he had 45; how many had he at first?

10. A horse was bought for $90 and sold for $122; what was the gain?

11. There are 93 scholars in 3 classes: 23 in the first, and 37 in the second; how many in the third?

12. My watch cost $23. I paid $5 for repairing it, and then sold it for $43; what was my gain?

13. From a cistern containing 100 gallons, 46 gallons were drawn at one time, and 38 at another. How many gallons remained in the cistern?

14. A boy gave his watch and $12 for 2 suits of clothes worth $36. What was the value of the watch?

WRITTEN EXERCISES.

1. A woman went shopping with $140 in her purse; and on her return she had only $65; how much did she spend? *Ans.* $75.

2. A cistern which holds 210 gallons, was filled in the morning, but at night there were but 50 gallons remaining. How many gallons had leaked out? *Ans.* 160 gallons.

3. In a certain village there were 587 men drafted for the army; but 296 were rejected by the surgeon. How many were taken? *Ans.* 291 men.

4. The less of two numbers is 6347, and their sum is 15813; what is the greater? *Ans.* 9466.

5. If Mrs. Andrew Johnson was 75 years old in 1880, in what year was she born? *Ans.* 1805.

6. The subtrahend is 919, and the minuend is 4813; what is the remainder? *Ans.* 3894.

7. A jeweller bought a watch for $96, and a chain for $38. He sold both for $235; how much did he gain? *Ans.* $101.

8. Two men purchased a horse; one paying 320 dollars, and the other 47 dollars less; how much did both pay? *Ans.* 593 dollars.

9. Arthur had 297 nuts in a basket. He put 85 in one bag and 79 in another; how many were left in the basket? *Ans.* 133 nuts.

10. I sold a farm for $201860, thereby gaining $90990; how much did I pay for it? *Ans.* $110870.

11. An auctioneer received furniture to the value of $7864, which he auctioned off in two lots, one for $4620.75, and the other for $3000; what was the loss on the furniture? *Ans:* $243.25.

MULTIPLICATION.

51. ***Multiplication*** is the process of taking one number as many times as there are units in another.

52. The ***Multiplicand*** is the number taken or multiplied.

53. The ***Multiplier*** is the number by which we multiply. It shows how many times the multiplicand is taken.

54. The ***Product*** is the number obtained by multiplying.

NOTE.—The multiplicand and the multiplier are the *Factors* of the product. Thus, 6 and 3 are the factors of the product 18.

55. The ***Sign of Multiplication*** is the oblique cross, ×, and is read *times,* or *multiplied by.* When placed between two numbers it denotes that their product is to be taken. Thus, 8 × 6 indicates that 8 is to be multiplied by 6, or that 6 is to be multiplied by 8.

MULTIPLICATION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108

56. **When the multiplier contains only one figure.**

1. Let it be required to multiply 895 by 7.

[illegible]—After writing the multiplier under the lowest order plicand and drawing a line, we begin to multiply at the right. 7 times 5 units are 35 units, which are equal to 3 tens and 5 units. We write the 5 in units' place, and reserve the 3 tens to be added to the product of the tens. Multiplying 9 tens by 7 we get 63 tens, which increased by the 3 tens reserved, give 66 tens, or 6 hundreds and 6 tens. We set down the 6 tens in tens' place, and reserve the 6 hundreds to add to the next product. 7 times 8 hundreds are 56 hundreds, plus the 6 hundreds reserved, make 62 hundreds, or 6 thousands and 2 hundreds. As we have no more orders to multiply, we put the 6 thousands and the 2 hundreds in their proper places. The resulting number, 6265, is the required product.

OPERATION.

Multiplicand,	895
Multiplier,	7
Product,	6265

Proof.—If the product be the same when the multiplicand and the multiplier are interchanged, the work is correct.

WRITTEN EXERCISES.

Multiply:

2. 789 by 8.
3. 897 by 9.
4. 756 by 2.
5. 676 by 3.
6. 749 by 4.
7. 876 by 5.
8. 768 by 6.
9. 789 by 7.
10. 4284 by 8.
11. 3026 by 9.
12. 2008 by 7.
13. 8945 by 6.
14. 5327 by 5.
15. 24654 by 5.
16. 53407 by 6.
17. 23247 by 7.
18. 51847 by 8.
19. 57432 by 9.
20. 37476 by 6.
21. 578045 by 8.
22. 974834 by 9.
23. 375406 by 2.
24. 927454 by 5.
25. 905453 by 6.

CASE II.

57. When the multiplier contains more than one figure.

1. Find the product of 643 by 58.

OPERATION.

Multiplicand,	643	
Multiplier,	58	
1st partial product.	5144	= 643 × 8
2d partial product.	3215	= 643 × 50
Product,	37294	= 643 × 58

Solution.—Having written the multiplier under the multiplicand, so that units stand under units, tens under tens, etc., we begin at the right hand and multiply all the parts of the multiplicand, successively, by each term of the multiplier. Since

58 is equal to 5 tens and 8 units, 58 times 643 must be equal to 5 tens times 643 plus 8 units times 643. Having found 8 units times 643 by the rule under Case I., to be 5144, we set it down as the first partial product. To find 5 tens times 643, we begin by multiplying 3 units by 5 tens, which gives 15 tens, or 150 units. Omitting the naught for units' place, we set down the 5 tens in tens' place and carry the 1 hundred to the next product. By multiplying the remaining terms of the multiplicand by the 5 tens of the multiplier, and carrying as before, we obtain the second partial product, 32150; which is equal to 5 tens or 50 times 643. Now, by adding the partial products, 5144 and 32150, we find the entire product, 37294. In like manner we may find the product of any two numbers, being careful to place the first figure of each partial product under the corresponding figure of the multiplier.

WRITTEN EXERCISES.

Multiply:

2. 386 by 31.
3. 750 by 52.
4. 674 by 74.
5. 456 by 69.
6. 687 by 95.
7. 984 by 88.
8. 6307 by 44.
9. 9555 by 75.
10. 8616 by 63.
11. 4201 by 99.
12. 1187 by 55.
13. 7648 by 325.
14. 2185 by 204.
15. 9311 by 679.
16. 8209 by 123.
17. 83672 by 840.
18. 21016 by 189.
19. 37804 by 506.
20. 765407 by 257.
21. 709857 by 340.
22. 650074 by 457.
23. 834765 by 518.
24. 8759 by 3618.
25. 3269 by 2943.
26. 3587 by 7352.
27. 9145 by 8024.
28. 7352 by 3806.
29. 5747 by 1789.
30. 4382 by 2849.
31. 26937 by 2582.
32. 84132 by 4016.
33. 73041 by 7364.
34. 12345 by 6789.

35. 290008 by 12009.
36. 604082 by 351756.
37. 615138 by 254715.
38. 800739 by 700032.
39. 9103256 by 823906.
40. 9400631 by 746201.
41. 2935760 by 342007.
42. 657984854 by 518.
43. 696007453 by 673.
44. 973895676 by 1087.
45. 475795834 by 2076.
46. 785747827 by 3476.
47. 807954369 by 4637.
48. 584476854 by 5728.

CONTRACTIONS.

58. ***Contractions*** in multiplication are abbreviated methods of multiplying.

1. Multiply 428 by 32.

SOLUTION.—The factors of 32 are 8 and 4, or 4 × 4 × 2, or 2 × 2 × 2 × 2 × 2. Hence, 428 × 32 = 428 × 8 × 4. 428 × 8 = 3424; and 3424 × 4 = 13696.

WRITTEN EXERCISES.

2. 582 × 36.
3. 635 × 72.
4. 976 × 54.
5. 783 × 24.
6. 694 × 32.
7. 641 × 16.
8. 1745 × 63.
9. 2695 × 45.
10. 2146 × 96.
11. 1794 × 144.
12. 2563 × 132.
13. 3586 × 120.
14. 62816 × 121.
15. 70432 × 108.
16. 53006 × 99.
17. 42315 × 84.
18. 37640 × 42.
19. 45607 × 132.

59. Annexing a cipher to a number moves each of its digits one place to the left, thus converting units into tens, tens into hundreds, etc.; which is the same as multiplying the number by 10. Hence, to multiply a number by 10, we annex one cipher; to multiply it by 100, we annex two ciphers; and so on.

ILLUSTRATIONS.

1. Multiply 35 by 240. *2.* Multiply 3500 by 240.

OPERATION.

$$\begin{array}{l} 35 \\ 24\,|\,0 \\ \hline 140 \\ 70 \\ \hline 8400 \end{array}$$

OPERATION.

$$\begin{array}{l} 35\,|\,00 \\ 24\,|\,0 \\ \hline 140 \\ 70 \\ \hline 840000 \end{array}$$

WRITTEN EXERCISES.

Multiply:

3. 8607 by 10.
4. 8038 by 100.
5. 7415 by 100.
6. 1706 by 200.
7. 1679 by 300.
8. 4164 by 120.
9. 1482 by 350.
10. 98900 by 24000.
11. 63000 by 15000.
12. 70090 by 17500.
13. 10007 by 20060.
14. 90061 by 300000.
15. 60009 by 504000.
16. 791000 by 11700.

UNITED STATES CURRENCY.

60. When one of the factors contains *cents,* or *dollars* and *cents,* multiply as in simple numbers. Point off two places from the right, in the product, and prefix the sign $.

1. What cost 42 tons of coal at $5.75 a ton?

OPERATION.

$5.75
42
1150
2300
$241.50

SOLUTION.—If one ton cost $5.75, 42 tons will cost 42 times $5.75, which is $241.50. Since the multiplicand contains cents, we must point off two places in the product.

WRITTEN EXERCISES.

2. What will 25 hogsheads of sugar cost at $75.80 a hogshead?

3. Multiply $3502.40 by 162. By 250. By 584.

4. Multiply $4715.45 by 2405. By 810. By 675.

5. Multiply $3010.25 by 16. By 198. By 487.

6. What will be the cost of 144 yards of broadcloth at $3.75 a yard?

7. What is the value of 408 acres of land at $172.75 an acre?

8. Mr. Spencer bought 40 acres of land at $300 an acre, and 75 acres at $252.50 an acre. He sold the whole at $435 an acre: how much did he gain?

9. If a yard of cloth cost $2.25, what will 160 pieces, each containing 45 yards, cost?

10. How much is lost by selling at $4.25 a barrel, 1500 barrels of flour that cost $5.50 a barrel?

MENTAL EXERCISES.

1. What will 75 pounds of sugar cost at 12 cents a lb.?

2. At 3 cents apiece what will 80 oranges cost?

3. What cost 137 pine-apples at 10 cents apiece?

4. How much will 9 yards of cloth cost at $2.45 a yard?

5. What will a man spend in 5 days at the rate of $1.50 a day?

6. From a piece of cloth containing 93 yards, a tailor cut 6 suits of 10 yards each; how many yards had he left?

7. How many pounds in 6 loads of wheat, each load containing 20 bushels of 50 pounds each?

8. In an orchard there are 9 rows of trees, and 65 trees in each row; how many trees in the orchard?

9. A gentleman owing $176, gave in payment 6 shares of railroad stock worth $25 a share. How much does he still owe?

10. If I buy 8 barrels of pork at $15 a barrel, and sell it for $135, how much will I gain?

11. A boy earns $4.25 a week and pays 50 cents each day for his board, etc.; how much does he save in the week?

12. I sold 48 cords of wood at $5 a cord, and received in payment $180 cash, and the remainder in cloth. What was the value of the cloth?

13. How many pounds of cotton in 5 bales, each containing 350 pounds?

14. If one quart of peanuts cost 7 cents, what will 36 quarts cost?

15. What will a man pay for 5 pounds of veal at 12 cents a pound, and 9 pounds of beef at 18 cents a pound?

16. A grocer having 35 melons sold 16 of them, and then bought 4 times as many as he sold; how many had he then?

17. At $8 a barrel, what will be the cost of 5 loads, each containing 13 barrels?

WRITTEN EXERCISES.

1. A fortune of $125000 is to be so divided that each of four sons will receive $5825; each of two daughters $3570; and the widow the remainder. How much will the widow receive? *Ans.* $94560.

2. I bought 986 acres of cleared land at $64 an acre, and 60 acres of woodland at $36 an acre. I then sold the whole at $50 an acre. Did I gain or lose, and how much? *Ans.* Lost $12964.

3. If an acre of city land cost $1425, and were sold for $1320, what would be the loss on 37 acres? *Ans.* $3885.

4. A merchant bought at one time 576 pounds of coffee, and at another 425 pounds; what did it cost at 32 cents a pound? *Ans.* $320.32.

5. A man sold 8 umbrellas that were worth 85 cents each, for $1.30 each. How much more than their real value did he receive? *Ans.* $3.60.

6. A grazier bought 92 calves at $14 each, and sold 63 of them at $16 each, and the remainder at $13 each. What did he gain by the bargain? *Ans.* $97.

7. What amount of money will enable me to give $85.75 to each of 354 laborers? *Ans.* $30355.50.

8. A drover bought 140 head of cattle in Texas at $36 a head, and sent them to New York at an expense of $350.65. They were then sold at a profit of $1002.50. What did he receive for the drove? *Ans.* $6393.15.

9. Bought 3 pieces of sheeting containing respectively 25, 30, and 47 yards. The first cost 14 cents, the second 15, and the third 12 cents a yard. What was the entire cost? *Ans.* $13.64.

10. A farmer sold 74 barrels of apples at $5 a barrel, 44 barrels of potatoes at $3.50 a barrel, and 32 tons of hay at $17.75 a ton. What was the amount received?

Ans. $1092.

11. The President's salary is $50000, and his cabinet consisting of 7 members, receive each $8000; what is the total salary of the President and his cabinet?

Ans. $106000.

12. Two men are 860 miles apart. If they travel toward each other, one at the rate of 45 miles a day and the other 24, how far apart will they be in 12 days?

Ans. 32 miles.

13. In 1861 the national debt of Great Britain was $133.55 for each person, and the population, 29344788. What was the national debt?

Ans. $3,918,996,437.40.

14. A man worth $15840, received a legacy of $6000. He spent $4900 in traveling, and donated $3875 to a college; how much money has he remaining?

Ans. $13065.

DIVISION.

61. ***Division*** is the process of finding how many times one number is contained in another of the same kind.

62. The ***Dividend*** is the number to be divided.

63. The ***Divisor*** is the number by which we divide.

64. The ***Quotient*** is the result of the division. It shows how many times the dividend contains the divisor.

65. The ***Sign of Division***, ÷, read *divided by*, when placed between two numbers, indicates that the number before it is to be divided by the one after it. Thus, the expression, 63 ÷ 7, is read 63 divided by 7.

SHORT DIVISION.

1. Let it be required to divide 32540 by 5.

OPERATION.
Dividend
Divisor 5) 32540
Quotient 6508

SOLUTION.—Having written the divisor at the left of the dividend, with a curved line between them, we begin at the left to divide the different parts of the dividend by the divisor. Since 5 is not contained in 3 tens of thousands any tens of thousands times, we divide 32 thousands by 5. This gives 6 thousands for a quotient and 2 thousands for a remainder. We write the 6 thousands under the thousands, and to the remaining 2 thousands we annex the next term, which is 5 hundreds. 2 thousands and 5 hundreds are equal to 25 hundreds. 5 is contained in 25 hundreds 5 hundreds times. Since there is no remainder, and since 4 is less than 5, there are no tens in the quotient. We, therefore, write 0 in the place of tens, and annex the following rm to the 4 tens, making 40 units. Dividing 40 units by 5 we obain 8 units, which we place in the quotient under units. Hence the umber 6508, being the sum of all the partial quotients obtained by ividing the parts of the dividend by the divisor, is the required quotient.

PROOF.—Multiply the quotient by the divisor, and to the product add the remainder. If the result be equal to the dividend, the work is correct.

WRITTEN EXERCISES.

Divide:

2. 605 by 5.
3. 786 by 6.
4. 612 by 7.
5. 872 by 8.
6. 206 by 9.
7. 432536 by 8.
8. 405252 by 9.
9. 344688 by 2.
10. 478354 by 3.
11. 107425 by 4.
12. 741107808 by 7.
13. 456904117 by 8.
14. 741018207 by 9.
15. 746784320 by 5.
16. 402084009 by 6.

LONG DIVISION.

1. Let it be required to divide 50289 by 372.

OPERATION.

```
Divisor. Dividend. Quotient.
  372  ) 50289  ( 135
         372
         ----
         1308
         1116
         ----
          1929
          1860
          ----
Remainder,  69
```

SOLUTION.—Since 372 is not contained in 5 tens of thousands, or in 50 thousands any thousands times, there are no thousands in the quotient. Annexing the next term, 2 hundreds, we have 502 hundreds. 372 is contained in 502 hundreds 1 hundred times, with a remainder. Write the 1 hundred in the quotient, and multiply the divisor by it, subtracting the product from the 502 hundreds. This gives for remainder 130 hundreds; to which we annex the next term, 8 tens, making 1308 tens for the next partial dividend. The quotient of 1308 tens by 372 is greater than 3 and less than 4; hence there are 3 tens in the quotient. Multiplying 372 by 3 tens, we have 1116 tens, and this taken from 1308 tens, leaves 192 tens; to which we annex the next term, 9 units, making 1929 units. 372 is contained in 1929 units 5 times, with a remainder. Writing the 5 units in the quotient, and multiplying and subtracting as before, we obtain the remainder, 69. Hence the quotient is 1 hundred, 3 tens, and 5 units, or 135, with a remainder of 69.

NOTE.—The method of proving long division is the same as that for short division.

WRITTEN EXERCISES.

Divide:

2. 879420 by 64.
3. 796425 by 75.
4. 492630 by 66.
5. 843255 by 87.
6. 943079045 by 535.
7. 633576807 by 493.
8. 476958421 by 207.
9. 897435804 by 356.
10. 116418209 by 627.
11. 896047040 by 770.
12. 431651423 by 921.
13. 427850017 by 999.
14. 1316740006 by 814.
15. 9205639995 by 275.
16. 234827206 by 1047.
17. 764106347 by 5943.
18. 541224807 by 6481.
19. 684124206 by 5398.
20. 541307650 by 4765.
21. 984356401 by 2034.
22. 456087654 by 75979.
23. 864207979 by 79672.
24. 7464804605 by 296489.
25. 1700095084 by 346845.
26. 9467807008 by 374817.

66. ***Contractions*** in division are abbreviated methods of dividing.

1. Divide 1404 by 36, using factors.

OPERATION.

$36 = 9 \times 4$
$4\,)\ 1404$
$9\,)\ 351$
39

SOLUTION.—The factors of 36 are 9 and 4. By dividing the dividend by 4, and the quotient thus obtained by 9, we obtain the true quotient. 1404 ÷ 4 = 351 *fours.* 351 *fours* ÷ 9 = 39 ***thirty-sixes.*** Hence 36 is contained in 1404, 39 times.

2. Divide 1162 by 42, using the factors 3, 7, and 2.

OPERATION.

3) 1162
7) 387..................1
2) 55........$2 \times 3 = 6$
27...$1 \times 3 \times 7 = 21$
Rem. 28

SOLUTION.—Since the factors of 42 are 3, 7, and 2, we divide by each in order. $1162 \div 3 = 387$ *threes* and a remainder of 1 *unit*; 387 *threes* $\div 7 = 55$ *twenty-ones* and a remainder of 2 *threes*, or 6 *units;* 55 twenty-ones $\div 2 = 27$ *forty-twos* and a remainder of 1 *twenty-one*, or $1 \times 3 \times 7 = 21$ *units*. Add the partial remainders, $1 + 6 + 21 = 28$. Hence, 42 is contained in 1162, 27 times, with a remainder of 28.

3. Divide 3782 by 400.

OPERATION.

4 | 00) 37 | 82
$9\frac{182}{400}$

SOLUTION.—Cut off the two ciphers in the divisor, and the two terms at the right of the dividend; then four *hundreds* are contained in 37 *hundreds*, 9 times, with a remainder of 1 *hundred;* this 1 *hundred* + 82, the portion cut off = 182. Thus, $3782 \div 400 = 9\frac{182}{400}$.

WRITTEN EXERCISES.

Divide:

4. 16848 by 36.
5. 15225 by 125.
6. 26656 by 49.
7. 79645 by 56.
8. 90374 by 124.

9. 45113 by 200.
10. 37341 by 520.
11. 98465 by 2000.
12. 15640 by 860.
13. 72034 by 27000.

UNITED STATES CURRENCY.

67. Reduce the dividend to cents if necessary, and divide as in simple numbers. The quotient will be the answer in cents; which may be reduced to dollars and cents by placing the separating point two places from the right.

When both dividend and divisor are in currency, reduce each to cents if necessary, and divide as in simple numbers. The quotient will be the required number.

ILLUSTRATIONS.

1. Divide $402 equally among 25 men.

2. How many barrels of flour at $8.25 per barrel, can be purchased for $528?

```
         1.                              2.

25 ) $402.00 ( $16.08      $8.25 ) $528.00 ( 64 barrels.
     25                             4950
     ---                            ----
     152                             3300
     150                             3300
     ----                            ----
       200
       200
       ---
```

Divide:

3. $793.52 by 56.
4. $3924.72 by 237.
5. $30220 by 540.
6. $75612.58 by 784.
7. $299.64 by 33 cents
8. $45.99 by 21 cents.
9. $45038.75 by $18.75
10. $1104.00 by $36.80.

◆

MENTAL REVIEW.

1. If 25 sheep cost $200, what will one cost?

2. A farmer bought a wagon for $99, paying in hay at $11 a ton; how many tons of hay did he require?

3. For $216 how many barrels of pork can be bought at $8 a barrel? At $4? At $16?

4. If 9 apples are worth 135 chestnuts, how many chestnuts is 1 apple worth? 15 apples?

5. If 147 cents be divided among 7 boys, how many cents does each receive?

6. If 5 pounds of coffee cost 45 cents, what will 8 pounds cost? 12? 15? 20?

7. What will be the cost of 16 dozen of eggs, if 9 dozen cost 90 cents? 54 cents? 72 cents?

8. If 24 apples are worth 8 oranges, how many apples are 18 oranges worth? 25 oranges?

9. If 8 barrels of flour cost $72, what will 4 barrels cost? 11 barrels? 6 barrels? 22 barrels?

10. A boy earned 75 cents in 15 days; how much will he earn in 20 days? 25 days? 9 days?

11. A man travels 30 miles in 6 hours; how far will he travel in 14 hours? 16 hours? 32 hours?

12. How many melons can be bought for 75 cents, at the rate of 4 for 60 cents? 3 for 25 cents?

13. What must I pay for 12 pounds of beef, if 5 pounds cost 80 cents? For 8 pounds?

14. If 7 pounds of sugar cost 91 cents, what will 13 pounds cost? 10 pounds? 8 pounds?

15. How many hundredweight of hay can I buy for $65, if 15 hundredweight cost $75?

16. If 9 pigs are worth $63, what will 19 pigs cost?

17. If a man require 100 days to build 25 rods of fence how long will it take him to build 17 rods?

18. If for 8 meals 8 soldiers have 64 pounds of meat equally divided among them, how many pounds will each receive for one meal?

19. If a steamship make 54 miles in 3 hours, how long will it take her to make 90 miles? 126 miles? 72 miles?

20. If 8 benches are sufficient to accommodate 120 boys, how many benches will be required for 450 boys?

21. If 130 walnuts be equally divided among 13 boys, how many will 5 of them receive? 9? 11?

22. What will 13 sheep cost at the rate of 50 sheep for $150? 16 sheep? 35 sheep?

23. Paid $35 for 5 barrels of flour. At the same price, what will 25 barrels cost?

24. How many barrels of flour at $9 a barrel, are equal in value to 27 yards of cloth at $3 a yard?

25. How many pears at 5 cents apiece, can I get for 85 apples worth 2 cents apiece?

26. If 6 men dig a ditch in 35 days, how many men will do it in 7 days? In 14 days? In 42 days?

27. If it take 15 yards of cloth to make 3 suits of clothes, how many yards will 9 suits require?

28. When peaches are sold for 50 cents a basket, how many baskets must be given in exchange for 5 cords of wood at $3 a cord?

29. If 7 firkins of butter are worth $84, how many barrels of cider at $3 a barrel, will 4 firkins of butter buy?

30. How many quarts of milk at 5 cents a quart, will pay for 15 pounds of sugar at 8 cents a pound?

◆

WRITTEN REVIEW.

1. In how many days will a ship sail 7950 miles, if it sail 150 miles a day? *Ans.* 53 days.

2. A man owing $8637, gives in payment 32 acres of land valued at $26 per acre, and $3500 in cash; how much remains unpaid? *Ans.* $4305.

3. I bought 14 barrels of pork at $12 a barrel, and sold it for $154; how much did I gain or lose per barrel?
Ans. Lost $1 per barrel.

4. A lady paid $300 for a piano, $56 for a sofa, and $18 for an easy chair; how much less than three times $200 did all cost? *Ans.* $226.

5. How many gallons of molasses at 45 cents a gallon, will pay for 86490 pounds of butter at 25 cents a pound? *Ans.* 48050 gallons.

6. John and Peter started from home together and travelled in opposite directions. John travelled at the rate of 19 miles a day, and Peter at the rate of 27 miles a day; how far apart were they in 37 days? *Ans.* 1702 miles.

7. A farmer sold 356 oxen at the rate of 6 for $192. What did he receive for them? *Ans.* $11392.

8. How much can a person earn in 3 years, if he earn $120 in 24 days? *Ans.* $5475.

9. An estate of $50607 was left to a family of 4 brothers and 9 sisters. The brothers having given up their share to the sisters, how much did each of the latter receive? *Ans.* $5623.

10. A hogshead containing 63 gallons of molasses was bought for 54 cents a gallon. 6 gallons leaked out, and the rest was sold at a gain of 8 cents per gallon. What was the selling price? *Ans.* $35.34.

11. A farmer wishes to exchange 200 bushels of oats at 36 cents a bushel, for flour at $8 a barrel. How many barrels will he receive? *Ans.* 9 barrels.

12. If 34 persons consume 167 pounds of flour every day, how long will 13694 pounds last them? *Ans.* 82 days.

13. What number subtracted 18 times from 97632, will leave 2232 as a remainder? *Ans.* 5300.

14. A gentleman whose salary is $1800, saves $340 every year. What are his daily expenses? *Ans.* $4.

15. The sum of two numbers is 564, and their difference is 126; what is their product? *Ans.* 75555.

16. What number must be multiplied by 102 to produce 110670? *Ans.* 1085.

17. From what number can 251 be subtracted 405 times and leave a remainder of 73? *Ans.* 101728.

18. How long would it take a railway train to go from New York to San Francisco, 3300 miles, at the rate of 20 miles an hour? *Ans.* 165 hours.

19. What will 5 gross of eggs cost at 29 cents a dozen (12 dozen = 1 gross)? *Ans.* $17.40.

20. How many horses worth $124 apiece can be bought for 32 oxen at $74 each, 45 cows at $57 each, 235 sheep at $4 each, and $4543 cash? *Ans.* 84 horses.

21. A's income is 6 times B's; B's income is 4 times C's; and C's income is $2425. What is their entire income? *Ans.* $70325.

22. A man travels due north for 9 days at the rate of 49 miles a day. He then returns on his path at the rate of 36 miles a day; how far is he from the starting point at the end of 12 days? *Ans.* 9 miles.

23. How many yards of muslin at 15 cents a yard must be given in exchange for 75 bushels of oats at 47 cents a bushel? *Ans.* 235 yards.

24. At the rate of 37 miles a day, how long will it take a man to walk from Chicago to San Francisco, a distance of 2442 miles? *Ans.* 66 days.

25. A man having $7603, paid out all but $1735 in 9 weeks. What was the average amount paid out each week? *Ans.* $652.

26. Sold for $6817, a quantity of wood that cost $4312, thereby gaining $5 a cord. What was the cost per cord? *Ans.* $12.

27. Bought 54 barrels of flour for $324. For what must it be sold a barrel to gain $162? *Ans.* $9.

PROPERTIES OF NUMBERS.

68. The ***Properties of Numbers*** are those qualities or elements which necessarily belong to numbers.

69. An ***Even Number*** is a number that is exactly divisible by 2.

All numbers whose unit figure is 0, 2, 4, 6, or 8, are even.

70. An ***Odd Number*** is a number that is not exactly divisible by 2.

All numbers whose unit figure is 1, 3, 5, 7, or 9, are odd.

71. A ***Prime Number*** is a number that has no integral factors except unity and itself; as 2, 3, 7, 13, 23, 31.

72. A ***Composite Number*** is a number that has other integral factors besides unity and itself. Thus, 24 is a composite number; because 24 = 8 × 3, 6 × 4, or 12 × 2.

73. The ***Factors*** of a composite number are the numbers which, when multiplied together, will produce it; as, 3 and 5 are the factors of 15.

FACTORING.

74. ***Factoring*** is the operation of separating a number into its integral factors.

ORAL EXERCISES.

1. What are the factors of 6? Of 8? Of 15? Of 39? Of 84? Of 63? Of 75?

2. What are the prime numbers from 10 to 43? From 27 to 95? From 37 to 51?

3. What are the composite numbers from 23 to 51? From 47 to 64?

4. Mention the odd numbers between 19 and 31; 73 and 87; 45 and 61.

5. Mention the even numbers between 201 and 237; 506 and 532.

6. What are the prime factors of 21? Of 51? Of 95?

7. What are the composite factors of 24? Of 54?

8. Give both the prime and composite factors of 42.

9. What prime factors are common to 39 and 78?

10. What composite factors are common to 24 and 96?

11. What factors are common to the sum and difference of 60 and 120?

12. What is the greatest composite factor of 120?

OPERATION OF FACTORING.

1. Let it be required to separate 420 into its prime factors.

OPERATION.

$$\begin{array}{r|r} 2 & 420 \\ \hline 2 & 210 \\ \hline 3 & 105 \\ \hline 5 & 35 \\ \hline & 7 \end{array}$$

SOLUTION.—Since the prime number is even, we divide it by 2, the least prime factor; and the quotient we also divide by 2, which gives an odd number for result. This result we divide by 3, its least prime factor. We observe that the last quotient, 35, has for its prime factors 5 and 7. Having divided by 5, we obtain 7 for result. As 7 is not divisible, it must be a prime factor of the given number. Hence, the divisors 2, 2, 3, 5, and the last quotient 7, are all the prime factors or divisors of 420; that is, 420 = 2 × 2 × 3 × 5 × 7.

WRITTEN EXERCISES.

Resolve the following numbers into their prime factors:

2. 24.
3. 65.
4. 40.
5. 88.
6. 48.
7. 36.
8. 64.
9. 84.
10. 72.
11. 78.
12. 60.
13. 99.
14. 112.
15. 128.
16. 154.
17. 165.
18. 360.
19. 160.
20. 378.
21. 450.
22. 486.
23. 504.
24. 726.
25. 1155.

GREATEST COMMON DIVISOR.

75. An ***Exact Divisor*** of a number is a number that will divide it without a remainder. Thus, 4 is an exact divisor of 32.

76. ***The Greatest Common Divisor*** of two or more numbers is the greatest number that exactly divides each of them. Thus, 6 is the greatest common divisor of 18 and 42.

NOTE.—A common divisor of the divisor and the dividend, is also a divisor of the remainder; and a common divisor of the remainder and divisor, is also a common divisor of the dividend. Thus, the common divisor 2, of the divisor 6 and the dividend 32, is also a common divisor of the remainder 2.

ORAL EXERCISES.

1. Name two exact divisors of 6. Of 8. Of 12. Of 32. Of 50.

2. Name three exact divisors of 48. Of 16. Of 96. Of 42.

3. What number is an exact divisor of 52? Of 91? Of 49?

4. Name all the exact divisors of 40. Of 25. Of 63.

5. What divisors are common to 20 and 30? To 35 and 105? To 24 and 60?

6. What are the prime factors of 60? Of 80? Of 50? Of 70? Of 25? Of 45?

7. What prime factors are common to 30 and 50?

8. What is the greatest common divisor of 12 and 18? Of 24 and 48?

9. What two numbers will exactly divide 20 and 40? Their sum and difference?

10. What is the greatest common divisor of 26, 52, and 91?

11. Find the greatest common divisor of the sum and difference of 36 and 90.

12. What is the greatest divisor common to 99 and 121?

CASE I.

77. When the numbers are small and can be easily factored.

In the first method of this case, we find the common factors and take their product.

1. Find the greatest common divisor of 48, 72, and 120.

OPERATION.

$$\begin{array}{r|rrr} 2 & 48, & 72, & 120 \\ \hline 2 & 24, & 36, & 60 \\ \hline 2 & 12, & 18, & 30 \\ \hline 3 & 6, & 9, & 15 \\ \hline & 2, & 3, & 5 \end{array}$$

SOLUTION.—Having written the numbers as in the margin, we observe that 2 is a factor of each. It is, therefore, a factor of the greatest common divisor.

The quotients may also be divided by 2 as a common factor. The second set of quotients also have 2 for a common factor. We divide the next set of quotients by their common factor 3, and thus obtain for the last quotients, the prime numbers 2, 3, 5. Hence, the numbers, 2, 2, 2, and 3, being all the common factors, their product, 24, will be the greatest common divisor.

WRITTEN EXERCISES.

What is the greatest common divisor of :

2. 10, 30, and 40?
3. 24, 32, and 56?
4. 16, 48, and 64?
5. 25, 65, and 75?
6. 120, 360, and 480?
7. 272, 288, and 396?
8. 96, 144, and 504?
9. 504, 546, and 648?
10. 640, 320, and 480?
11. 392, 504, and 648?
12. 180, 320, and 342?
13. 900, 468, and 342?
14. 126, 210, and 252?
15. 144, 720, and 360?
16. 165, 242, and 726?
17. 330, 420, and 560?
18. 924, 1210, and 1683? *Ans.* 11.
19. 1365, 2340, and 3549? *Ans.* 39.
20. 1287, 2079, 4488, and 6384? *Ans.* 11.
21. 2898, 3588, 3795, and 3910? *Ans.* 23.
22. 7020, 8316, 9126, and 10773? *Ans.* 27.

The second method in this case is to resolve the numbers into their prime factors, and take the product of their common factors.

1. Find the greatest common divisor of 48, 72, and 120.

OPERATION.

$$48 = 2 \times 2 \times 2 \times 2 \times 3$$
$$72 = 2 \times 2 \times 2 \times 3 \times 3$$
$$120 = 2 \times 2 \times 2 \times 3 \times 5$$
$$2 \times 2 \times 2 \times 3 = 24$$

SOLUTION.—The factors of 48 are 2, 2, 2, 2, and 3; the factors of 72 are 2, 2, 2, 3, and 3; and the factors of 120 are 2, 2, 2, 3, and 5. We see that 2, 2, 2, and 3 are all the prime factors common to the three given numbers; hence, their product, 24, is the greatest common divisor.

WRITTEN EXERCISES.

Find the greatest common divisor of :

2. 6, 9, and 27.
3. 8, 16, and 24.
4. 10, 40, and 60.
5. 24, 32, and 56.
6. 16, 48, and 64.
7. 25, 65, and 75.
8. 36, 48, and 72.
9. 88, 99, and 154.
10. 24, 40, and 64.
11. 65, 78, and 117.

CASE II.

78. When the numbers are large and cann readily factored.

1. Find the greatest common divisor of 316 and 66

OPERATION.

```
316 ) 664 ( 2
      632
      ———
       32 ) 316 ( 9
            288
            ———
             28 ) 32 ( 1
                  28
                  —
                   4 ) 28 ( 7
                       28
                       —
```

SOLUTION.—We divide 6 316, the divisor 316, by the re der 32, the divisor 32, by t mainder 28, and the divisor the remainder 4, and obtain mainder. Now, the greatest mon divisor of 316 and 664, i the greatest common divisor and 316 (**76**); and the gr common divisor of 32 and also the greatest common d of 28 and 32; but the greatest mon divisor of 28 and 32 hence, the greatest common divisor of 316 and 664 is 4.

WRITTEN EXERCISES.

What is the greatest common divisor of :

2. 320 and 496 ?
3. 972 and 624?
4. 420, 240, and 540?
5. 324, 672, and 720?
6. 1203, 2406, and 4812 ?
7. 1728 and 1872?
8. 364 and 584 ?
9. 826, 1372, and 41
10. 2040, 8476, and 4
11. 2192, 1456, and 1

LEAST COMMON MULTIPLE.

79. ***The Least Common Multiple*** of tw more numbers is the least number which is a multi each of them. Thus, 24, 48, and 72, are each a com multiple of 2, 3, and 4 ; but 12 is their least com multiple.

ORAL EXERCISES.

1. What numbers between 3 and 41 are exactly divi by 5 ? By 6 ? By 8 ? By 9 ?

2. What numbers less than 60 are exactly divisibl 8? By 5? By 9? By 12?

3. What multiple is common to 5 and 6? To 9 and 11? o 8 and 3? To 7 and 12? To 6 and 7?

4. What is the least common multiple of 3 and 7? Of and 9? Of 4 and 11? Of 9 and 4?

5. What is the least common multiple of 4 and 6? Of and 12? Of 8 and 12? Of 24 and 84?

6. Name at least four numbers of which 3 and 4 are ctors. 2 and 5. 6 and 7. 2 and 3.

7. By what prime factors can 63 be divided? 56? 48?

8. Name at least three common multiples of 8 and 9.

9. Mention three numbers of which 2 and 3 are factors.

10. Find at least four common multiples of 3 and 5.

11. What is the least common multiple of 6 and 12? 4 and 10? Of 5 and 30?

WRITTEN EXERCISES.

1. Find the least common multiple of 16, 20, and 30.

OPERATION.

$= 2 \times 2 \times 2 \times 2$

$= 2 \times 2 \times 5$

$= 2 \times 3 \times 5$

SOLUTION.—The least common multiple cannot be less than the largest number, 30, since it must contain 30. Hence, it must contain all the prime factors of 30, which are 2, 3, and 5. But the least common ltiple of 30, must also contain all the prime factors of each of the er numbers. Now, since the prime factor 2, of 30, is common to 20 and 16, we omit it, and annex the factors 2, 2, and 2 to se of 30. The series 2, 3, 5, 2, 2, and 2 are all the prime factors the given numbers, and their product, 240, is the least common ltiple of the given numbers.

SECOND METHOD.

Find the least common multiple of 16, 20, and 30.

PERATION.

16, 20, 30

8, 10, 15

4, 5, 15

4, 1, 3

SOLUTION.—Having placed the numbers in a horizontal line, we see that 2 is a factor of each of them. It is, therefore, a factor of the least common multiple. Continuing to divide the quotient by the factors 2 and 5, we obtain the *numbers*, 4, 1, and 3, which, being *prime* to one another, are the last quotients. The factors of the given numbers are 2, 2, 5, 4, and 3; and their product, 240, is the least common multiple.

Find the least common multiple of:

2. 60, 40, and 90.
3. 24, 56, and 84.
4. 17, 59, and 85.
5. 48, 72, and 81.
6. 240, 120, and 260.
7. 960, 690, and 810.
8. 30, 42, and 72.
9. 32, 40, and 25.
10. 60, 12, and 63.
11. 56, 49, and 77.
12. 54, 108, and 207 . . . . *Ans.* 2484
13. 27, 216, and 414 . . . . " 4968.
14. 291, 485, 679. . . . . " 10185.
15. 33, 39, 77, 91, 143. . . " 3003.
16. 34, 38, 85, 95, 323. . . " 3230.
17. 87, 203, 231, 319. . . . " 6699.
18. 51, 69, 85, 115, 391. . " 5865.
19. 14, 34, 35, 85, 119. . . " 1190.
20. 102, 114, 255, 285, 232 " 9690.
21. 12, 18, 30, 48, 16 . . . *Ans.* 720.

CANCELLATION.

80. ***Cancellation*** is the process of abridging operations in division by rejecting equal factors from both dividend and divisor.

1. Divide 32 × 96 by 12 × 16.

1st OPERATION.

$$\frac{32 \times 96}{12 \times 16} = \frac{\cancel{8} \times \cancel{4} \times 16 \times \cancel{3} \times \cancel{2}}{\cancel{3} \times \cancel{4} \times \cancel{8} \times \cancel{2}} = 16$$

SOLUTION.—Having written the numbers that constitute the dividend above a line and those that constitute the divisor below it, resolve each of the numbers into its respective factors. After rejecting equal factors from both dividend and divisor there remains the factor 16 in the dividend. Hence, the quotient is 16.

2D OPERATION.

$$\frac{\overset{2}{\cancel{32}} \times \overset{8}{\cancel{96}}}{\cancel{12} \times \cancel{16}} = 16$$

SOLUTION.—Since 12 will divide both 96 in the dividend and 12 in the divisor, reject 12 as a factor from both, retaining the factor 8 in the dividend. In the same manner reject the factor 16 from both dividend and divisor, leaving the factor 2 in the dividend. There remain the factors 8 and 2 in the dividend, whose product is the required quotient.

WRITTEN EXERCISES.

2. Divide 24 × 14 × 16 by 6 × 7 × 32. *Ans.* 4.

3. Divide 20 × 48 × 10 by 96 × 8 × 4. *Ans.* $3\frac{1}{8}$.

4. Divide 25 × 120 × 10 by 80 × 40 × 4. *Ans.* $2\frac{11}{32}$.

5. Divide 45 × 63 × 28 by 9 × 126 × 56. *Ans.* $1\frac{1}{4}$.

6. Divide 60 × 39 × 15 by 120 × 78 × 30. *Ans.* $\frac{1}{8}$.

7. Divide 27×35×48×58 by 29×56×18×30. *Ans.* 4.

8. Divide 24×36×69×75 by 23×50×72×45. *Ans.* 1.

9. Divide 13×19×17×20 by 17×260×4×361.

10. Divide 81×95×44×72 by 19×18×15×27. *Ans.* 176.

11. Divide 11×15×19×27×28 by 12×22×45×49×57. *Ans.* $\frac{1}{14}$.

12. Divide 13×25×51×48×44 by 17×32×55×65×81. *Ans.* $\frac{2}{9}$.

13. Divide 15×14×18×38×4×65 by 8×13×19×21×25×36. *Ans.* 1.

14. Divide 35×25×64×57×63×78×99 by 18×21×27×50×52×55×76. *Ans.* 14.

WRITTEN REVIEW.

1. If it require 15 men, working 10 hours a day for 24 days, to do a certain work, how many men, working 8 hours a day for 50 days, would be required to do the same work?

2. Find three numbers less than 150, which are multiples of 16 and 24.

3. How many tubs of butter, each containing 45 pounds, worth 30 cents a pound, will cost the same as 25 barrels of sugar, each containing 300 pounds, at 10 cents a pound?

4. The minuend is the L. C. M. of 108 and 144; the subtrahend is the G. C. D. of 108 and 144; find the remainder. *Ans.* 396.

5. I can buy a number of cows at $60 each, sheep at $12 each, and hogs at $18 each; what is the least sum of money that will pay for them?

6. Find the least number which, when divided by 5, 8, 9, 7, or 4, will leave 3 as a remainder in each case.

7. There are three tanks holding, respectively, 1500, 2550, and 3000 gallons. What is the capacity of the largest vessel that will exactly fill them if emptied into them an exact number of times?

8. Divide 39 × 5 × 84 × 36 by 75 × 26 × 96 × 7, and multiply the quotient by 20.

9. How many dozen of eggs, at 33 cents per dozen, can be exchanged for 495 pounds of sugar at 12 cents a pound?

10. Find the length of the longest curb-stones that will exactly fit each of three strips of sidewalk, the first of which is 546 feet long, the second 588 feet, and the third 1134 feet.

11. What three numbers between 170 and 850 have the same G. C. D. as these numbers?

12. In a certain house there are four rooms whose widths are, respectively, 51, 45, 30, and 33 feet. What is the width of the widest carpeting that will exactly fit each of them?

13. The L. C. M. of 95, 5, 7, and another number prime to each of them, is 5320; what is the other number?

14. What is the least sum of money with which I can buy oxen at $16 each, or cows at $36 each, or horses at $45 each, and spend all my money each time? *Ans.* $720.

15. Find the G. C. D. and also the L. C. M. of 44, 99, 33, and 77.

16. How many days' work, at $1.25 a day, will pay for 65 bushels of corn, at $.50 a bushel? *Ans.* 26.

17. The G. C. D. of two numbers is 15, and their L. C. M. is 90. Find the numbers.

FRACTIONS.

81. A ***Fraction*** is one or more of the equal parts of a unit.

82. Fractions may be divided into two classes: *common fractions* and *decimal fractions.*

83. A ***Common Fraction*** is one in which the unit is divided into any number of equal parts.

84. A *common fraction* is expressed by two numbers, one written above the other, with a short line between them; as, $\frac{4}{5}$ expresses 4 fifths.

85. The ***Denominator*** of a fraction denotes the number of equal parts into which the unit is divided. It is written below the line.

86. The ***Numerator*** of a fraction denotes the number of equal parts taken. It is written above the line.

87. Common Fractions are divided into three classes: *Simple, Compound,* and *Complex.*

88. A ***Simple Fraction*** is a fraction having a single integral numerator and denominator; as, $\frac{5}{8}$; $\frac{3}{4}$.

89. A ***Proper Fraction*** is a simple fraction whose value is less than a unit; as, $\frac{7}{8}$; $\frac{2}{13}$.

90. An ***Improper Fraction*** is a simple fraction whose value is equal to or greater than a unit; as, $\frac{5}{4}$; $\frac{13}{9}$.

91. A ***Compound Fraction*** is a fraction of a fraction; as, $\frac{3}{4}$ of $\frac{5}{6}$; $\frac{6}{7}$ of $\frac{8}{9}$.

92. A ***Complex Fraction*** is a fraction whose numerator, or denominator, or both, are fractional; as,
$\frac{6}{\frac{3}{4}}$; $\frac{\frac{7}{8} \text{ of } \frac{2}{3}}{\frac{9}{10} \text{ of } \frac{11}{12}}$; $\frac{\frac{4}{7} \times 1\frac{3}{4} \times \frac{5}{8}}{2\frac{1}{2} \times \frac{4}{5} \times 4\frac{1}{3}}$.

93. A ***Mixed Number*** is a whole number and a fraction united; as, $9\frac{1}{2}$; $7\frac{3}{4}$.

EXERCISES.

Read the following fractions and mixed numbers:

1. $\frac{1}{2}$.
2. $\frac{1}{3}$.
3. $\frac{1}{4}$.
4. $\frac{1}{5}$.
5. $\frac{2}{3}$.
6. $\frac{3}{4}$.
7. $\frac{1}{6}$.
8. $\frac{1}{7}$.
9. $\frac{2}{5}$.
10. $\frac{3}{7}$.
11. $\frac{6}{8}$.
12. $\frac{8}{9}$.
13. $\frac{4}{5}$.
14. $\frac{8}{8}$.
15. $\frac{7}{9}$.
16. $\frac{5}{6}$.
17. $\frac{9}{10}$.
18. $\frac{11}{12}$.
19. $\frac{5}{13}$.
20. $\frac{9}{14}$.
21. $\frac{7}{15}$.
22. $\frac{8}{11}$.
23. $\frac{9}{17}$.
24. $\frac{6}{19}$.
25. $\frac{4}{4}$.
26. $\frac{5}{3}$.
27. $\frac{7}{6}$.
28. $\frac{3}{3}$.
29. $\frac{8}{5}$.
30. $\frac{7}{7}$.
31. $\frac{9}{8}$.
32. $\frac{8}{7}$.
33. $\frac{15}{16}$.
34. $\frac{21}{25}$.
35. $\frac{16}{48}$.
36. $\frac{47}{62}$.
37. $\frac{19}{52}$.
38. $\frac{31}{84}$.
39. $\frac{22}{35}$.
40. $\frac{25}{49}$.
41. $\frac{73}{100}$.
42. $\frac{45}{203}$.
43. $\frac{127}{320}$.
44. $\frac{54}{182}$.
45. $\frac{75}{300}$.
46. $\frac{67}{150}$.
47. $\frac{99}{200}$.
48. $\frac{141}{420}$.
49. $1\frac{1}{2}$.
50. $3\frac{2}{3}$.
51. $8\frac{1}{4}$.
52. $5\frac{1}{2}$.
53. $7\frac{1}{5}$.
54. $9\frac{2}{7}$.
55. $6\frac{7}{8}$.
56. $3\frac{4}{5}$.

Express by figures:

One third, Two eighths, Five sevenths, Two fifths, Four ninths, Five eighths, Three fourths, One ninth, Four elevenths, Five sixths, Seven eighths, Seven fifteenths, Nine twentieths, Three tenths, Six thirteenths, Nine tenths, Eleven thirty-thirds, Sixteen seventy-fifths, Fifty-one sixty-ninths, Twenty-four sixty-sevenths, Thirteen fourteenths, Forty-nine fiftieths, Sixty-two eighty-fourths, Ninety one-hundredths, Eighty-six one hundred twentieths, Nine sevenths, Six sixths, Thirty-two elevenths, Eighty-four fortieths, Thirty-seven twenty-sixths, One hundred one eighty-fifths.

REDUCTION.

94. ***Reduction of Fractions*** is the operation of changing their form without altering their value.

CASE I.

95. To reduce fractions to higher or lower terms.

ORAL EXERCISES.

1. Reduce $\frac{4}{6}$ to its lowest terms.

SOLUTION.—Since a fraction is expressed in its lowest terms when its numerator and denominator are prime to each other, we divide the terms of the given fraction $\frac{4}{6}$, by their greatest common divisor 2, and obtain $\frac{2}{3}$ for result. This result, therefore, expresses the lowest terms of the given fraction.

2. Change $\frac{6}{8}$ to its lowest terms. $\frac{4}{8}$. $\frac{6}{12}$. $\frac{3}{6}$. $\frac{5}{10}$.

3. Change $\frac{16}{24}$ to its lowest terms. $\frac{4}{12}$. $\frac{5}{15}$. $\frac{6}{24}$. $\frac{10}{18}$.

4. Change to their lowest terms: $\frac{3}{9}$, $\frac{6}{10}$, $\frac{4}{8}$, $\frac{7}{21}$, $\frac{6}{18}$, $\frac{9}{6}$, $\frac{12}{8}$, $\frac{6}{4}$, $\frac{10}{24}$, $\frac{12}{36}$, $\frac{18}{30}$, $\frac{6}{42}$, $\frac{8}{10}$, $\frac{13}{26}$, $\frac{42}{49}$, $\frac{21}{24}$.

5. Change $\frac{1}{3}$ to its higher terms by 2, by 3, by 6.

SOLUTION.—Since the value of a fraction is not changed by multiplying both numerator and denominator by the same number, we multiply the terms of the given fraction $\frac{1}{3}$ by 2, and obtain $\frac{2}{6}$ for result. This result expresses higher terms of the given fraction. In like manner we may multiply the terms of the given fraction by any number.

6. Change tc equivalent higher terms: $\frac{1}{4}$, $\frac{2}{3}$, $\frac{3}{7}$, $\frac{4}{5}$, $\frac{2}{11}$, $\frac{1}{7}$, $\frac{1}{6}$, $\frac{3}{87}$, $\frac{5}{61}$, $\frac{3}{4}$, $\frac{7}{81}$.

7. Change $\frac{4}{5}$ to an equivalent fraction having 40 for denominator.

8. Reduce $\frac{5}{7}$ to a fraction having 56 for a denominator.

9. Reduce $\frac{15}{16}$ to forty-eighths. $\frac{4}{9}$ to eighty-firsts.

WRITTEN EXERCISES.

I.

Reduce the following to their lowest terms:

1. $\frac{48}{54} = 2)\ \frac{48}{54} = 3)\ \frac{24}{27} = \frac{8}{9}$, *Ans.*

2. $\frac{40}{150} = 10)\ \frac{40}{150} = \frac{4}{15}$, *Ans.*

NOTE.—When the terms of the fraction are large and cannot be readily factored, find the greatest common divisor before reducing Thus, to reduce $\frac{1261}{3298}$ to its lowest terms, divide both terms by their greatest common divisor, 97.

3. $\frac{21}{63}$.
4. $\frac{36}{40}$.
5. $\frac{18}{45}$.
6. $\frac{25}{60}$.
7. $\frac{48}{87}$.
8. $\frac{70}{96}$.
9. $\frac{15}{145}$.
10. $\frac{75}{120}$.
11. $\frac{17}{153}$.
12. $\frac{76}{285}$.
13. $\frac{91}{406}$.
14 $\frac{83}{257}$.
15. $\frac{72}{108}$.
16. $\frac{150}{240}$.
17. $\frac{648}{980}$.
18. $\frac{252}{404}$.
19. $\frac{84}{252}$.
20. $\frac{840}{980}$.
21. $\frac{609}{696}$.
22. $\frac{250}{1000}$.
23. $\frac{824}{1236}$.
24. $\frac{1555}{2799}$.
25. $\frac{262}{1441}$.
26. $\frac{1080}{1296}$.

II.

Find the value of :

1. $\frac{6\times8}{4\times18}$. *Ans.* $\frac{2}{3}$.

2. $\frac{9\times5}{20\times3}$. *Ans.* $\frac{3}{4}$.

3. $\frac{9\times8\times6}{16\times3\times5}$. *Ans.* $\frac{9}{5}$.

4. $\frac{3\times6\times7}{14\times3\times15}$. *Ans.* $\frac{1}{5}$.

5. $\frac{9\times6\times8}{12\times16\times18}$. *Ans.* $\frac{1}{8}$.

6. $\frac{3\times11\times4}{8\times12\times22}$. *Ans.* $\frac{1}{16}$.

7. $\frac{7\times8\times3}{15\times14\times7}$. *Ans.* $\frac{4}{35}$

8. $\frac{12\times9\times1}{13\times24\times18}$. *Ans.* $\frac{1}{52}$

9. $\frac{9\times7\times5\times13}{14\times1\times58\times18}$.

10. $\frac{4\times2\times15\times11}{9\times33\times8\times2}$.

11. $\frac{9\times5\times25\times28}{15\times3\times56\times40}$.

12. $\frac{11\times27\times13\times45\times16}{15\times26\times22\times54\times75}$.

13. $\frac{18\times20\times125\times36\times84}{60\times12\times96\times72\times120}$.

14. $\frac{35\times21\times65\times44\times250}{325\times90\times40\times36\times27}$

CASE II.

96. To reduce whole or mixed numbers to simple fractions.

ORAL EXERCISES.

1. In 2 units how many halves?

SOLUTION.—Since the parts of a unit which has been divided into 2 equal parts, are called halves, every unit must, therefore, contain 2 halves. Now, if one unit contains 2 halves, 2 units will contain twice as many halves as 1 unit, or twice 2 halves, which are 4 halves. Therefore, in 2 units there are 4 halves.

2. In 4 units how many fifths? Halves? Fourths?
3. How many sixths in 3? 8? 7? 10? 2? 12?
4. How many thirds in 4? 5? 9? 11? 7? 8?
5. In \$5 how many half dollars? Quarter dollars?
6. How many $\$\frac{1}{2}$ will buy a barrel of flour worth \$8?
7. Reduce 8 to fourths. 9 to sixths. 3 to halves.
8. Reduce 17 to thirds. 3 bushels to 7ths of bushels.
9. How many 9ths of yards in 3 yards? 5 yards?
10. How do you reduce a number to thirds? Eighths?
11. In $2\frac{1}{2}$ how many halves? $3\frac{1}{2}$? $5\frac{1}{2}$? $8\frac{1}{2}$? $12\frac{1}{2}$?
12. How many thirds in $2\frac{1}{3}$? $3\frac{1}{3}$? $5\frac{2}{3}$? $7\frac{2}{3}$? $10\frac{1}{3}$?
13. How many fifths in $3\frac{1}{5}$? $6\frac{2}{5}$? $2\frac{3}{5}$? $4\frac{4}{5}$? $7\frac{3}{5}$?
14. Reduce to improper fractions $4\frac{2}{3}$. $2\frac{4}{7}$. $4\frac{3}{11}$.
15. Reduce $2\frac{5}{6}$ to sixths. $7\frac{5}{8}$ to eighths. $13\frac{1}{3}$ to thirds.
16. In $8\frac{9}{10}$ how many tenths? $4\frac{3}{10}$? $15\frac{1}{10}$? $6\frac{1}{10}$?
17. How many 20ths in 8? 12? 25? 16?
18. Reduce $9\frac{1}{4}$ to an improper fraction. $15\frac{1}{6}$. $25\frac{1}{8}$.
19. How many 12ths in 30? 70? 80? 14? 21?
20. In 15 how many 8ths? 10ths? 9ths? 7ths?
21. Reduce $8\frac{8}{9}$ to an improper fraction. $2\frac{1}{12}$. $1\frac{5}{16}$.
22. Reduce $22\frac{1}{2}$ to an improper fraction. $33\frac{1}{3}$. $37\frac{1}{2}$.

WRITTEN EXERCISES.

1. Reduce $96\frac{6}{7}$ to sevenths.

OPERATION.

$$1 \text{ unit} = \frac{7}{7}$$
$$96 \text{ units} = \frac{7}{7} \times 96 = \frac{672}{7}$$
$$\frac{672}{7} + \frac{6}{7} = \frac{678}{7}$$

SOLUTION.—As there are 7 sevenths in 1 unit, in 96 units there will be 96 times as many sevenths as in 1 unit, or 96 times $\frac{7}{7}$, which are $\frac{672}{7}$. $\frac{672}{7}$ and $\frac{6}{7}$ make $\frac{678}{7}$. Therefore, $96\frac{6}{7}$ are equal to $\frac{678}{7}$. *Ans.*

2. Express 28 in the form of a fraction having for denominator 3; 7; 11; 9; 16; 19. *Ans.* to first, $\frac{84}{3}$.

3. 7 is equal to what fraction having 18 for denominator? *Ans.* $\frac{126}{18}$.

4. Express 27 as fifths. *Ans.* $\frac{135}{5}$.

5. How many tenths in 38 pounds? *Ans.* $\frac{380}{10}$ pounds.

6. In 42 yards how many $\frac{1}{36}$ yards? *Ans.* $\frac{1512}{36}$ yards.

7. Reduce 3, 27, 63, 18, 42, to 13ths. *Ans.* to first, $\frac{39}{13}$.

8. How many 4ths in 6, 8, 13, 25, 75, 67?

9. How many 3ds in 9, 11, 16, 42, 7, 8?

10. What fraction with 17 for denominator is equal to 9? 15? 20? 12? *Ans.* $\frac{153}{17}$, $\frac{272}{17}$, $\frac{340}{17}$, $\frac{204}{17}$.

11. Reduce $9\frac{7}{12}$ to twelfths. *Ans.* $\frac{115}{12}$.

12. Change $18\frac{17}{23}$ to twenty-thirds. *Ans.* $\frac{431}{23}$.

13. Reduce $37\frac{4}{9}$ to an improper fraction. *Ans.* $\frac{337}{9}$.

14. In $86\frac{3}{4}$ miles how many $\frac{1}{4}$ miles? *Ans.* $\frac{347}{4}$ miles.

15. In \$$50\frac{3}{4}$ how many $\frac{1}{4}$ dollars? *Ans.* $\frac{203}{4}$ dollars.

Reduce the following to improper fractions:

16. $8\frac{3}{7}$.
17. $7\frac{5}{13}$.
18. $9\frac{13}{17}$.
19. $42\frac{8}{11}$.
20. $33\frac{1}{3}$.
21. $31\frac{8}{19}$.
22. $106\frac{11}{14}$.
23. $273\frac{3}{5}$.
24. $75\frac{1}{2}$.
25. $67\frac{1}{2}$.
26. $66\frac{2}{3}$.
27. $99\frac{9}{11}$.
28. $30\frac{6}{25}$.
29. $472\frac{23}{75}$.
30. $682\frac{7}{125}$.
31. $16\frac{1}{2}$.
32. $35\frac{1}{4}$.
33. $150\frac{1}{6}$.
34. $200\frac{5}{8}$.
35. $109\frac{15}{16}$.

CASE III.

97. To reduce improper fractions to whole or mixed numbers.

ORAL EXERCISES.

1. How many bushels in $\frac{19}{4}$ bushels?

SOLUTION.—In 1 bushel, or 1 unit, there are $\frac{4}{4}$; and in $\frac{19}{4}$ bushels there are as many bushels as 4 are contained times in 19, which are $4\frac{3}{4}$ times. Therefore, $\frac{19}{4}$ bushels are equal to $4\frac{3}{4}$ bushels.

2. How many units in $\frac{8}{2}$ units? $\frac{6}{3}$ units? $\frac{10}{5}$ units?

3. Reduce to whole numbers: $\frac{8}{4}$, $\frac{16}{2}$, $\frac{18}{3}$, $\frac{27}{9}$, $\frac{36}{4}$.

4. In $\frac{36}{12}$ feet, how many feet? In $\frac{36}{6}$ feet?

5. Reduce to whole numbers: $\frac{48}{6}$ yards, $\frac{35}{7}$ feet.

6. How many tons in $\frac{23}{5}$ tons? $\frac{19}{4}$ tons? $\frac{19}{7}$ tons?

7. How many cents in $\frac{12}{5}$c.? $\frac{18}{7}$c.? $\frac{32}{6}$c.? $\frac{41}{8}$c.?

8. Reduce to mixed numbers: $\frac{13}{7}$, $\frac{25}{6}$, $\frac{42}{9}$, $\frac{71}{8}$, $\frac{53}{11}$, $\frac{33}{10}$.

WRITTEN EXERCISES.

1. Change $\frac{127}{13}$ to a whole or mixed number.

OPERATION.

$\frac{127}{13} = 127 \div 13 = 9\frac{10}{13}$, *Ans.*

SOLUTION.—Since there are $\frac{13}{13}$ in 1 unit, in $\frac{127}{13}$ there are as many units as 13 are contained times in 127, which are $9\frac{10}{13}$ times. Therefore, $\frac{127}{13} = 9\frac{10}{13}$.

Reduce the following to whole or mixed numbers:

2. $\frac{82}{3}$.
3. $\frac{59}{9}$.
4. $\frac{81}{13}$.
5. $\frac{74}{6}$.
6. $\frac{97}{8}$.
7. $\frac{64}{12}$.
8. $\frac{56}{9}$.
9. $\frac{35}{16}$.
10. $\frac{110}{11}$.
11. $\frac{208}{3}$.
12. $\frac{244}{121}$.
13. $\frac{204}{12}$.
14. $\frac{117}{7}$.
15. $\frac{234}{18}$.
16. $\frac{424}{106}$.
17. $\frac{948}{237}$.
18. $\frac{118}{9}$.
19. $\frac{206}{7}$.
20. $\frac{133}{5}$.
21. $\frac{277}{12}$.
22. $\frac{458}{39}$.
23. $\frac{271}{47}$.
24. $\frac{632}{78}$.
25. $\frac{802}{102}$.
26. $\frac{4912}{307}$.
27. $\frac{4704}{12}$.
28. $\frac{1248}{133}$.
29. $\frac{3721}{162}$.
30. $\frac{8002}{371}$.
31. $\frac{1803}{361}$.
32. $\frac{21762}{806}$.
33. $\frac{87107}{3705}$.

CASE IV.

98. To reduce fractions to equivalent fractions having a common denominator.

ORAL EXERCISES.

1. Reduce $\frac{1}{2}$, $\frac{3}{4}$, to a common denominator.

SOLUTION.—Since a common denominator of two or more fractions is a common multiple of their denominators, take the product of 2 and 4, which is 8, for the required denominator. Next, reduce the given fractions to others having 8 for a common denominator. $\frac{1}{2}$ is equal to $\frac{4}{8}$; and $\frac{3}{4}$ is equal to $\frac{6}{8}$. The given fractions are now said to be reduced to a common denominator.

2. $\frac{1}{3}$, $\frac{1}{6}$.
3. $\frac{1}{5}$, $\frac{1}{3}$.
4. $\frac{1}{2}$, $\frac{1}{3}$.
5. $\frac{5}{6}$, $\frac{1}{8}$.
6. $\frac{3}{7}$, $\frac{5}{9}$.
7. $\frac{2}{3}$, $\frac{3}{4}$.
8. $\frac{1}{8}$, $\frac{1}{6}$.
9. $\frac{2}{3}$, $\frac{2}{7}$.
10. $\frac{9}{11}$, $\frac{1}{2}$.
11. $\frac{9}{10}$, $\frac{6}{7}$.
12. $\frac{4}{5}$, $\frac{1}{2}$.
13. $\frac{2}{5}$, $\frac{2}{3}$.
14. $\frac{3}{8}$, $\frac{2}{5}$.
15. $\frac{5}{12}$, $\frac{2}{7}$.
16. $\frac{3}{17}$, $\frac{2}{5}$.

WRITTEN EXERCISES.

1. Reduce $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{7}{8}$, to their least common denominator.

OPERATION.

$$\frac{2}{3},\ \frac{5}{6},\ \frac{7}{8}.$$

$$\begin{array}{r|l} 3 & 3\ ,\ 6\ ,\ 8 \\ \hline 2 & 1\ ,\ 2\ ,\ 8 \\ \hline & 1\ ,\ 1\ ,\ 4 \end{array}$$

$$3 \times 2 \times 4 = 24, \text{ L. C. D.}$$

$$\frac{2}{3}\,\frac{\times}{\times}\,\frac{8}{8} = \frac{16}{24}$$

$$\frac{5}{6}\,\frac{\times}{\times}\,\frac{4}{4} = \frac{20}{24}$$

$$\frac{7}{8}\,\frac{\times}{\times}\,\frac{3}{3} = \frac{21}{24}$$

SOLUTION.—Having found the least common multiple of the given denominators to be 24, reduce each fraction to equivalent fractions having 24 for denominator.

To reduce $\frac{2}{3}$ to an equivalent fraction, having 24 for denominator, divide 24 by the denominator 3, and multiply both terms by the quotient, 8. $\frac{5}{6}$ and $\frac{7}{8}$ may be reduced in the same way.

Reduce to their least common denominator:

2. $\frac{2}{3}, \frac{4}{5}, \frac{7}{8}$.
3. $\frac{3}{8}, \frac{4}{6}, \frac{2}{5}$.
4. $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}$.
5. $\frac{5}{6}, \frac{8}{9}, \frac{6}{7}$.
6. $\frac{7}{9}, \frac{5}{8}, \frac{5}{6}$.
7. $\frac{4}{9}, \frac{1}{3}, \frac{3}{4}$.
8. $\frac{3}{5}, \frac{2}{7}, \frac{2}{3}$.
9. $\frac{1}{4}, \frac{2}{9}, \frac{3}{5}$.
10. $\frac{3}{4}, \frac{4}{11}, \frac{10}{22}$.
11. $\frac{3}{8}, \frac{7}{25}, \frac{5}{10}$.
12. $\frac{2}{3}, \frac{4}{15}, \frac{7}{20}$.
13. $\frac{1}{4}, \frac{15}{16}, \frac{9}{20}$.
14. $\frac{5}{9}, \frac{30}{54}, \frac{8}{27}$.
15. $\frac{11}{21}, \frac{16}{39}, \frac{6}{13}$.
16. $\frac{14}{18}, \frac{31}{36}, \frac{61}{72}$.
17. $\frac{5}{12}, \frac{7}{18}, \frac{11}{24}$.
18. $\frac{13}{18}, \frac{18}{20}, \frac{22}{36}, \frac{40}{60}$.
19. $\frac{3}{14}, \frac{26}{28}, \frac{28}{35}, \frac{17}{63}$.
20. $\frac{5}{12}, \frac{7}{36}, \frac{9}{48}, \frac{11}{72}$.
21. $\frac{7}{48}, \frac{17}{96}, \frac{3}{24}, \frac{1}{12}$.
22. $\frac{24}{75}, \frac{37}{56}, \frac{14}{15}, \frac{1}{28}$.
23. $\frac{13}{42}, \frac{8}{21}, \frac{16}{28}, \frac{9}{10}$.
24. $\frac{5}{12}, \frac{7}{15}, \frac{15}{16}, \frac{24}{25}$.
25. $\frac{7}{42}, \frac{71}{84}, \frac{101}{168}, \frac{93}{108}$.

ADDITION.

99. ***Addition of Fractions*** is the operation of finding the sum of two or more fractions.

ORAL EXERCISES.

1. What is the sum of $\frac{3}{4} + \frac{2}{4}$? $\frac{2}{3} + \frac{5}{6}$? $5\frac{1}{2} + 4\frac{3}{5}$?

SOLUTION.—Addition of fractions implies three operations: 1st to add fractions having a common denominator; 2d, to add fractions having different denominators; and 3d, to add mixed numbers. The first is performed by simply adding the numerators, for a new numerator, and keeping the common denominator. Thus, $\frac{3}{4} + \frac{2}{4} = \frac{5}{4} = 1\frac{1}{4}$, *Ans.* Secondly, before adding fractions of different denominators, it is necessary to reduce them to a common denominator. Thus, $\frac{2}{3} + \frac{5}{6} = \frac{4}{6} + \frac{5}{6} = \frac{9}{6} = 1\frac{3}{6} = 1\frac{1}{2}$, *Ans.*

In the third operation find the sum of the fractions and that of the integers separately, and then add the results. Thus, to add $5\frac{1}{2}$ and $4\frac{3}{5}$, we first find the sum of $\frac{3}{5}$ and $\frac{1}{2}$, which is $1\frac{1}{10}$. This added to the sum of the integers, 5 and 4, will make $10\frac{1}{10}$, *Ans.*

NOTES.—1. Mixed numbers may also be reduced to improper fractions and then added by the second operation.

2. The sum should always be expressed in its lowest terms; and if an improper fraction, should be reduced to a mixed number.

I.

2. $\frac{1}{2}+\frac{1}{2}$.
3. $\frac{1}{3}+\frac{1}{3}$.
4. $\frac{1}{4}+\frac{1}{4}$.
5. $\frac{2}{3}+\frac{1}{3}$.
6. $\frac{3}{4}+\frac{1}{4}$.
7. $\frac{3}{7}+\frac{2}{7}$.
8. $\frac{3}{8}+\frac{2}{8}$.
9. $\frac{3}{8}+\frac{1}{8}$.
10. $\frac{4}{9}+\frac{2}{9}$.
11. $\frac{1}{8}+\frac{5}{8}$.
12. $\frac{4}{11}+\frac{3}{11}$.
13. $\frac{6}{13}+\frac{5}{13}$.
14. $\frac{9}{20}+\frac{8}{20}$.
15. $\frac{1}{6}+\frac{2}{6}$.
16. $\frac{1}{2}+\frac{3}{4}$.
17. $\frac{2}{5}+\frac{1}{3}$.
18. $\frac{3}{4}+\frac{1}{3}$.
19. $\frac{2}{3}+\frac{1}{9}$.
20. $\frac{5}{8}+\frac{3}{4}$.
21. $\frac{1}{3}+\frac{1}{6}$.
22. $\frac{5}{12}+\frac{3}{12}$.
23. $\frac{3}{10}+\frac{5}{10}$.
24. $\frac{3}{14}+\frac{9}{14}$.
25. $\frac{2}{15}+\frac{4}{15}$.
26. $\frac{3}{16}+\frac{9}{16}$.
27. $\frac{5}{18}+\frac{7}{15}$.
28. $\frac{3}{7}+\frac{2}{5}$.
29. $\frac{6}{7}+\frac{3}{7}$.
30. $\frac{4}{9}+\frac{8}{9}$.
31. $\frac{8}{5}+\frac{6}{5}$.
32. $\frac{3}{4}+\frac{3}{4}$.
33. $\frac{2}{3}+\frac{4}{3}$.
34. $\frac{5}{6}+\frac{1}{12}$.
35. $\frac{6}{11}+\frac{1}{3}$.
36. $\frac{5}{12}+\frac{5}{6}$.
37. $\frac{5}{12}+\frac{9}{4}$.
38. $\frac{12}{13}+\frac{10}{13}$.
39. $\frac{14}{17}+\frac{10}{17}$.
40. $\frac{12}{25}+\frac{14}{5}$.
41. $\frac{8}{10}+\frac{4}{5}$.
42. $2\frac{1}{2}+\frac{1}{2}$.
43. $3\frac{1}{4}+\frac{1}{4}$.
44. $4\frac{2}{3}+1\frac{1}{3}$.
45. $2\frac{1}{7}+3\frac{4}{7}$.
46. $8\frac{4}{9}+4\frac{2}{9}$.
47. $5\frac{2}{5}+6\frac{4}{5}$.
48. $7\frac{7}{8}+3\frac{3}{8}$.
49. $3\frac{3}{4}+2\frac{1}{2}$.
50. $5\frac{2}{3}+4\frac{1}{6}$.
51. $9\frac{3}{8}+8\frac{1}{4}$.
52. $3\frac{1}{7}+4\frac{1}{2}$.
53. $6\frac{3}{5}+1\frac{1}{3}$

II.

1. James gave $\frac{1}{5}$ of a dollar for a ball and $\frac{2}{5}$ of a dollar for a pair of gloves; how much did he give for both?

2. A farmer sold $\frac{1}{6}$ of his potatoes to A, $\frac{1}{6}$ to B, and $\frac{3}{6}$ to C. What part of the whole did he sell to all?

3. Three boys hired a boat; John gave \$$\frac{1}{2}$, William \$$\frac{1}{4}$, and Henry \$$\frac{1}{4}$; what sum of money did they all pay?

4. A carpenter put up $\frac{3}{4}$ of a rod of fence, his son $\frac{1}{2}$ of a rod, and his apprentice $\frac{1}{2}$ of a rod; how long was the fence?

5. James gave \$$\frac{1}{4}$ for a circus ticket, \$$\frac{4}{5}$ for a straw hat, and had $\frac{1}{10}$ of a dollar remaining; how much had he at first?

6. Paid \$$8\frac{1}{2}$ for a bbl. of flour, \$$5\frac{1}{4}$ for a ton of coal, and \$$3\frac{1}{3}$ for a cord of wood; what was the amount of the bill?

7. How many yards of cloth will be required to make a pair of pants containing $1\frac{1}{2}$ yards, a vest $\frac{2}{3}$ of a yard, and a coat $2\frac{1}{6}$ yards?

8. Bought a mosaic table for \$$87\frac{3}{4}$ and sold it for \$$12\frac{1}{4}$ more than I paid for it; what was the selling price?

9. A man walked $13\frac{3}{4}$ miles on Monday and $14\frac{1}{6}$ miles on Tuesday; how far did he walk in the two days?

10. A boy earned \$$2\frac{1}{4}$ one week, \$2 the next week, and \$$3\frac{1}{2}$ the next week; how much did he earn in the 3 weeks?

11. What is the sum of \$$10\frac{1}{2}$, \$$5\frac{1}{2}$, \$$20\frac{1}{2}$, \$$14\frac{1}{2}$?

12. A man sold $5\frac{1}{2}$ bushels of cranberries at one time, $2\frac{1}{4}$ bushels at another, and $10\frac{3}{4}$ bushels at a third time. How many bushels did he sell in all?

13. A farm comprised a peach orchard of $6\frac{1}{8}$ acres, a wheat field of $12\frac{1}{2}$ acres, and a corn field of $9\frac{3}{4}$ acres. How many acres in the farm?

14. If a clerk copy $20\frac{1}{2}$ pages in one day, $30\frac{1}{3}$ pages in another, and $40\frac{5}{6}$ pages in another, how many pages will he copy in the three days?

WRITTEN EXERCISES.

1. Find the sum of $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{8}$, $\frac{1}{2}$, and $\frac{7}{12}$.

OPERATION.

$$\begin{array}{c|l} & \;\;24 \\ \cline{2-2} \frac{2}{3} & \;\;8 \times 2 = 16 \\ \frac{3}{4} & \;\;6 \times 3 = 18 \\ \frac{5}{8} & \;\;3 \times 5 = 15 \\ \frac{1}{2} & 12 \times 1 = 12 \\ \frac{7}{12} & \;\;2 \times 7 = 14 \\ \multicolumn{1}{c}{} & \qquad\qquad \overline{} \\ \multicolumn{1}{c}{} & \qquad\qquad \frac{75}{24} = 3\frac{1}{8} \end{array}$$

SOLUTION.—The least common denominator of the given fractions is 24. Having reduced them to 24ths, **Art. 98,** and taken the sum of their numerators, we find $\frac{75}{24}$, or $3\frac{1}{8}$ for result. Hence, the sum of $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{8}$, $\frac{1}{2}$, and $\frac{7}{12} = 3\frac{1}{8}$.

2. Find the sum of $4\frac{2}{9}$, $\frac{7}{8}$, and $9\frac{5}{6}$.

OPERATION.

$$\begin{array}{r|c|rcl}
 & & \multicolumn{3}{c}{72} \\ \cline{3-3}
4 & \frac{2}{9} & 8 \times 2 & = & 16 \\
 & \frac{7}{8} & 9 \times 7 & = & 63 \\
9 & \frac{5}{6} & 12 \times 5 & = & 60 \\ \cline{1-1}\cline{5-5}
13 & & & & \frac{139}{72} = 1\frac{67}{72}
\end{array}$$

$$13 + 1\frac{67}{72} = 14\frac{67}{72}.$$

SOLUTION.—Having separated the integers from the fractions, reduce the latter to equivalent fractions having a common denominator. Find their sum as in the first example, and add it to the sum of the integers. The result will be the answer required.

Find the sum of:

3. $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}$.
4. $\frac{1}{5}+\frac{1}{4}+\frac{1}{6}$.
5. $\frac{2}{7}+\frac{1}{8}+\frac{1}{2}$.
6. $\frac{4}{7}+\frac{3}{8}+\frac{2}{9}$.
7. $\frac{7}{8}+\frac{3}{4}+\frac{3}{5}$.
8. $\frac{6}{7}+\frac{5}{9}+\frac{1}{3}$.
9. $\frac{5}{6}+\frac{5}{8}+\frac{1}{9}$.
10. $\frac{4}{4}+\frac{1}{7}+\frac{1}{6}$.
11. $\frac{3}{7}+\frac{4}{9}+\frac{8}{21}$.
12. $\frac{4}{5}+\frac{1}{2}+\frac{9}{10}$.
13. $\frac{8}{9}+\frac{1}{3}+\frac{5}{36}$
14. $\frac{6}{7}+\frac{5}{6}+\frac{3}{14}$.
15. $\frac{5}{8}+\frac{2}{3}+\frac{13}{24}$.
16. $\frac{5}{6}+\frac{3}{4}+\frac{19}{20}$.
17. $\frac{1}{8}+\frac{7}{5}+\frac{27}{64}$.
18. $\frac{5}{9}+\frac{2}{3}+\frac{3}{11}$.
19. $\frac{5}{6}+\frac{8}{11}+\frac{1}{2}+\frac{5}{12}$.
20. $\frac{4}{9}+\frac{7}{36}+\frac{2}{5}+\frac{1}{18}$.
21. $\frac{16}{25}+\frac{7}{8}+\frac{1}{10}+\frac{3}{4}$.
22. $\frac{6}{7}+\frac{6}{35}+\frac{1}{5}+\frac{4}{25}$.
23. $\frac{1}{2}+\frac{4}{13}+\frac{1}{3}+\frac{2}{17}$.
24. $\frac{8}{15}+\frac{3}{7}+\frac{2}{9}+\frac{17}{50}$.
25. $\frac{14}{21}+\frac{19}{35}+\frac{1}{7}+\frac{8}{9}$.
26. $\frac{11}{12}+\frac{5}{24}+\frac{41}{48}+\frac{3}{5}$.

27. $\frac{5}{18}+\frac{1}{4}+\frac{7}{9}+\frac{25}{36}+\frac{3}{12}+\frac{1}{2}$.	*Ans.* $2\frac{3}{4}$.
28. $\frac{7}{16}+\frac{2}{5}+\frac{17}{25}+\frac{1}{10}+\frac{2}{3}+\frac{7}{8}$.	*Ans.* $3\frac{191}{1200}$
29. $\frac{19}{45}+\frac{1}{9}+\frac{3}{3}+\frac{47}{55}+\frac{2}{11}+\frac{1}{3}+\frac{7}{10}$.	*Ans.* $3\frac{199}{330}$.
30. $\frac{16}{33}+\frac{9}{11}+\frac{21}{22}+\frac{4}{3}+\frac{1}{2}+\frac{1}{4}+\frac{14}{15}$	*Ans.* $5\frac{181}{660}$.
31. $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{4}{5}+\frac{5}{6}+\frac{6}{7}+\frac{7}{8}+\frac{8}{9}+\frac{9}{10}$.	*Ans.* $7\frac{179}{2520}$.

II.

Add the following:

32. $5\frac{1}{3}$, $6\frac{1}{4}$, $1\frac{4}{5}$.
33. $9\frac{4}{5}$, $5\frac{1}{6}$, $3\frac{9}{10}$.
34. $3\frac{9}{11}$, $7\frac{19}{21}$, $10\frac{1}{2}$.
35. $15\frac{3}{8}$, $19\frac{11}{14}$, $5\frac{6}{7}$.
36. $6\frac{1}{3}$, $9\frac{11}{12}$, $4\frac{5}{16}$, $5\frac{7}{48}$.
37. $3\frac{1}{2}$, $4\frac{1}{4}$, $7\frac{1}{9}$, $9\frac{1}{6}$, $2\frac{5}{8}$.
38. $1\frac{1}{6}$, $3\frac{7}{27}$, $15\frac{1}{9}$, $23\frac{8}{54}$.
39. $45\frac{2}{3}$, $63\frac{3}{5}$, $5\frac{7}{8}$, $73\frac{1}{2}$.
40. $\frac{3}{7}$, $\frac{18}{63}$, $\frac{1}{9}$, $17\frac{8}{21}$, $6\frac{1}{3}$.
41. 45, $2\frac{15}{48}$, $9\frac{3}{16}$, $1\frac{3}{4}$.
42. $1\frac{2}{5}$, $9\frac{7}{12}$, $17\frac{8}{15}$, $18\frac{7}{30}$.
43. $2\frac{9}{16}$, $12\frac{1}{8}$, $3\frac{9}{32}$, $21\frac{3}{4}$.

44. $\frac{41}{35}$, $\frac{11}{20}$, $19\frac{3}{4}$, $68\frac{1}{5}$.

45. $\frac{14}{17}$, $5\frac{5}{12}$, $8\frac{16}{51}$, $7\frac{3}{17}$.

46. $9\frac{8}{39}$, $4\frac{1}{13}$, $\frac{18}{78}$, $4\frac{1}{6}$.

47. $\frac{74}{145}$, $6\frac{1}{9}$, $\frac{7}{15}$, $36\frac{1}{3}$.

48. $\frac{16}{35}$, $1\frac{19}{70}$, $2\frac{5}{7}$, $29\frac{9}{10}$.

49. $13\frac{1}{10}$, $1\frac{57}{100}$, $20\frac{131}{1000}$.

50. $6\frac{1}{4}$, $19\frac{17}{24}$, $50\frac{19}{48}$, $8\frac{1}{3}$.

51. $185\frac{1}{63}$, $9\frac{1}{2}$, $16\frac{7}{9}$, $2\frac{13}{81}$.

MENTAL EXERCISES.

1. Joseph bought a pair of skates for $\frac{5}{8}$ of a dollar, a pair of mittens for $\frac{4}{7}$ of a dollar, and a strap for $\frac{3}{4}$ of a dollar; what was the cost of the whole?

SOLUTION.—If the skates cost $\frac{5}{8}$ of a dollar, the mittens $\frac{4}{7}$, and the strap $\frac{3}{4}$, all together must cost the sum of $\frac{5}{8}+\frac{4}{7}+\frac{3}{4}$; which is $\frac{109}{56} = 1\frac{53}{56}$ dollars.

2. A farmer sold $\frac{1}{4}$ of his grain to one man, $\frac{2}{7}$ to another, and $\frac{3}{14}$ to another; what part of his grain did he sell to the three men together?

3. Jane paid $3\frac{1}{2}$ dollars for a dress, $2\frac{2}{3}$ for a pair of shoes, and $\frac{1}{6}$ for some ribbon; what did she pay for all?

4. Thomas saved $\frac{3}{4}$ of a dollar one week, $\frac{2}{3}$ of a dollar the next, $2\frac{1}{3}$ the next, and $5\frac{1}{2}$ the fourth week; how much did he save in the four weeks?

5. Three men bought a cow: Mr. Jones paid \$$15\frac{5}{6}$, Mr. Brown \$$24\frac{7}{24}$, and Mr. Smith \$$12\frac{1}{2}$; what was the cost of the cow?

6. Henry bought a penknife for $76\frac{7}{12}$ cents, and sold it for $23\frac{4}{5}$ more than it cost. What did he sell it for?

7. If it take $2\frac{1}{3}$ yards of cloth for a coat, $2\frac{1}{4}$ yards for a pair of pantaloons, and $\frac{5}{7}$ of a yard for a vest, how many yards will be required for the whole suit?

8. A man dug $\frac{1}{5}$ of a rod of a ditch the first hour, $\frac{4}{5}$ of a rod the second, $\frac{1}{9}$ the third, and $\frac{11}{12}$ the fourth; how many rods did he dig in the 4 hours?

9. How many pounds of candy in 4 packages containing respectively, $6\frac{1}{2}$, $5\frac{1}{4}$, 17, and $8\frac{9}{10}$ pounds?

10. If coal is worth $\$8\frac{7}{24}$ a ton, and wood is worth $\$6\frac{5}{12}$ a cord, what will be the cost of 2 tons of coal and 1 cord of wood?

11. Having $15\frac{9}{10}$ barrels of flour, I bought $3\frac{3}{20}$ barrels more. How many barrels had I then?

12. If Joseph walk $6\frac{3}{4}$ miles in the forenoon, and $4\frac{1}{5}$ miles in the afternoon, how many miles will he walk in two days at the same rate?

WRITTEN EXERCISES.

1. A gentleman travelled $56\frac{1}{6}$ miles in one day, $63\frac{3}{4}$ the next, $45\frac{1}{3}$ the third, and $20\frac{4}{9}$ the fourth. How far did he travel in all? *Ans.* $185\frac{25}{36}$ miles.

2. A boy earned $\$5\frac{3}{4}$ the first week, $\$6\frac{1}{8}$ the second week, $\$4\frac{7}{16}$ the third, $\$8\frac{1}{2}$ the fourth, $\$6\frac{7}{8}$ the fifth, and $\$2\frac{1}{4}$ the sixth. How much did he earn in the 6 weeks?

Ans. $\$33\frac{15}{16}$.

3. A farm contains $32\frac{1}{4}$ acres of meadow land, $24\frac{5}{6}$ acres of plough land, $72\frac{3}{5}$ acres of pasture land, and $81\frac{9}{10}$ acres of wood land; how many acres in the farm?

Ans. $211\frac{7}{12}$ acres.

4. How many pounds of raisins in 4 boxes containing respectively, $14\frac{5}{12}$ pounds, $27\frac{8}{9}$ pounds, $12\frac{14}{15}$ pounds, and $37\frac{9}{12}$ pounds? *Ans.* $92\frac{89}{90}$ pounds.

5. Three cheeses weigh respectively, $18\frac{2}{3}$, $64\frac{11}{12}$, and $14\frac{6}{16}$ pounds. What is their entire weight?

Ans. $97\frac{23}{24}$ pounds.

6. From a certain quantity of cloth, I cut three pieces, the first containing $124\frac{2}{9}$ yards, the second $234\frac{7}{12}$ yards, and the third $50\frac{15}{36}$ yards. How many yards in the three pieces?

Ans. $409\frac{2}{9}$ yards.

7. A has $\$150\frac{140}{150}$; B has $\$99\frac{24}{25}$; and C has $\$124\frac{1}{15}$. How much have they all? *Ans.* $\$375\frac{19}{25}$.

8. I sold 2 tubs of butter each containing $254\frac{5}{14}$ pounds, and 1 tub containing $96\frac{6}{7}$ pounds. How many pounds did I sell? *Ans.* $605\frac{4}{7}$ pounds.

9. Bought a ham for $\$1\frac{1}{3}$, a barrel of flour for $\$9\frac{4}{5}$, and some other articles for $\$15\frac{7}{8}$; what was the amount of the bill? *Ans.* $\$27\frac{1}{120}$.

10. What quantity of molasses in 4 casks, containing severally, $56\frac{1}{6}$, $24\frac{1}{3}$, $41\frac{8}{9}$, and $69\frac{2}{3}$ gallons?

Ans. $192\frac{1}{18}$ gallons.

11. One-fourth of an army was killed, $\frac{3}{8}$ of them taken prisoners, $\frac{1}{6}$ of them wounded, and the rest remained unhurt. What part of the army was rendered unfit for service? *Ans.* $\frac{19}{24}$.

12. I gave $\$2\frac{1}{5}$ for a knife, $\$1\frac{5}{6}$ for a pocket-book, $\$24\frac{1}{3}$ for a watch, and $\$9\frac{8}{12}$ for a watch-chain, and then found that I had $\$28\frac{9}{12}$ left. How much had I at first?

Ans. $\$66\frac{47}{60}$.

13. A poor person received $\frac{7}{8}$ of a dollar from one person, $\frac{15}{16}$ of a dollar from another, $2\frac{3}{48}$ dollars from a third. How much did he receive from all?

14. After spending $\$\frac{2}{5}$, $\$\frac{3}{10}$, $\$\frac{7}{8}$, and $\$\frac{2}{3}$, I had $\$9\frac{1}{2}$ left. How much had I at first?

SUBTRACTION.

100. ***Subtraction of Fractions*** is the process of finding the difference between two fractions.

ORAL EXERCISES.

What is the value of:

1. $\frac{3}{5}-\frac{2}{5}$? $\frac{3}{4}-\frac{1}{2}$? $3\frac{2}{3}-1\frac{1}{4}$?

SOLUTION.—When the fractions have a common denominator, we simply take the difference between their numerators, and keep the same denominator; thus, $\frac{3}{5}-\frac{2}{5}=\frac{1}{5}$, *Ans.*

Fractions having different denominators, must be reduced to equivalent fractions with a common denominator before subtracting; as $\frac{3}{4}-\frac{1}{2}=\frac{3}{4}-\frac{2}{4}=\frac{1}{4}$, *Ans.*

To find the difference between two mixed numbers, we subtract the fractions and the integers separately and then add the results. Thus, the difference between $3\frac{2}{3}$ and $1\frac{1}{4}$ is found as follows: $\frac{2}{3}-\frac{1}{4}=\frac{8}{12}-\frac{3}{12}=\frac{5}{12}$; and $3-1=2$. Adding $\frac{5}{12}$ to the integer 2, we obtain $2\frac{5}{12}$ for the result. Mixed numbers, when small, may also be subtracted by reducing them to improper fractions having a common denominator.

I.

2. $\frac{6}{7}-\frac{3}{7}$.
3. $\frac{8}{9}-\frac{1}{9}$.
4. $\frac{1}{2}-\frac{1}{2}$.
5. $\frac{2}{3}-\frac{1}{3}$.
6. $\frac{7}{8}-\frac{3}{8}$.
7. $\frac{3}{4}-\frac{1}{4}$.
8. $\frac{5}{11}-\frac{3}{11}$.
9. $\frac{6}{13}-\frac{1}{13}$.
10. $\frac{20}{21}-\frac{8}{21}$.
11. $\frac{5}{6}-\frac{1}{3}$.
12. $\frac{7}{8}-\frac{2}{4}$.
13. $\frac{4}{9}-\frac{1}{3}$.
14. $\frac{4}{7}-\frac{1}{2}$.
15. $\frac{3}{5}-\frac{1}{2}$.
16. $\frac{8}{3}-\frac{7}{4}$.
17. $\frac{11}{12}-\frac{5}{12}$.
18. $\frac{4}{18}-\frac{1}{18}$.
19. $\frac{18}{25}-\frac{3}{25}$.
20. $\frac{9}{16}-\frac{3}{8}$.
21. $\frac{8}{11}-\frac{2}{3}$.
22. $\frac{7}{15}-\frac{1}{5}$.
23. $\frac{8}{13}-\frac{1}{4}$.
24. $\frac{13}{2}-\frac{7}{4}$
25. $\frac{11}{7}-\frac{2}{3}$
26. $\frac{6}{5}-\frac{5}{15}$.
27. $\frac{8}{9}-\frac{2}{6}$.
28. $\frac{27}{15}-$ [illegible]
29. $4\frac{6}{8}-1\frac{3}{8}$.
30. $8\frac{6}{7}-2\frac{1}{7}$.
31. $9\frac{3}{4}-2\frac{1}{2}$.
32. $3\frac{7}{8}-2\frac{3}{4}$.
33. $5\frac{1}{2}-4\frac{1}{3}$.
34. $8\frac{3}{5}-2\frac{1}{4}$.
35. $4\frac{1}{7}-2\frac{1}{8}$.
36. $6\frac{1}{4}-5\frac{2}{5}$.
37. $8\frac{1}{3}-6\frac{1}{5}$.

II.

1. Joseph had \$$\frac{3}{4}$ and gave away \$$\frac{1}{2}$; how much had he left?

2. Lucy paid \$$\frac{4}{5}$ for a Geography and \$$\frac{3}{7}$ for a Grammar; how much more did the Geography cost than the Grammar?

3. A man owning a mill sold $\frac{5}{18}$ of it; what part does he still own?

4. Philip bought $2\frac{1}{2}$ pounds of candy and eat $1\frac{1}{4}$ pounds; how much had he left?

5. A girl having \$$7\frac{3}{5}$ loses \$$3\frac{1}{5}$; how much has she remaining?

6. A farmer owning $6\frac{1}{4}$ acres of land sells $2\frac{1}{4}$ acres; how many acres has he remaining?

7. A man having \$$10\frac{2}{3}$, paid \$$5\frac{1}{4}$ for a ton of coal, and \$$2\frac{3}{4}$ for a cord of wood ; how much had he left?

8. From $\frac{7}{10}$ of a dollar William spent $\frac{3}{10}$ of a dollar. What part of a dollar had he left?

WRITTEN EXERCISES.

1. Find the difference between $\frac{5}{9}$ and $\frac{3}{8}$.

OPERATION.

$\frac{5}{9} - \frac{3}{8} = \frac{13}{72}$

$$\begin{array}{r|l} & 72 \\ \hline \frac{5}{9} & 8 \times 5 = 40 \\ \frac{3}{8} & 9 \times 3 = \underline{27} \\ & \qquad\qquad \frac{13}{72} \end{array}$$

SOLUTION.—Reducing the given fractions to equivalent fractions having the least common denominator, we get $\frac{40}{72}$ and $\frac{27}{72}$. Then, $\frac{27}{72}$ from $\frac{40}{72}$ leave $\frac{13}{72}$.

2. From $157\frac{3}{5}$ take $85\frac{5}{6}$.

OPERATION.

$$\begin{array}{r|l} & 30 \\ \hline 157\frac{3}{5} & 6 \times 3 = 18 \\ \underline{85\frac{5}{6}} & 5 \times 5 = 25 \\ 71\frac{23}{30} & \qquad\qquad \frac{23}{30} \end{array}$$

SOLUTION.—Having reduced the fractions to equivalent fractions having the least common denominator, we find that $\frac{25}{30}$ cannot be taken from $\frac{18}{30}$. We, therefore, take $1 = \frac{30}{30}$, from 157 and add it to $\frac{18}{30}$, which gives $\frac{48}{30}$. Now, $\frac{25}{30}$ from $\frac{48}{30}$ leave $\frac{23}{30}$, and 85 from 156 leave 71. Hence, the required difference is $71\frac{23}{30}$.

Find the value of :

3. $\frac{5}{9}-\frac{1}{3}$.
4. $\frac{6}{7}-\frac{4}{5}$.
5. $\frac{2}{3}-\frac{5}{9}$.
6. $\frac{5}{6}-\frac{2}{3}$.
7. $\frac{8}{9}-\frac{3}{4}$.
8. $\frac{7}{8}-\frac{6}{7}$.
9. $\frac{4}{5}-\frac{1}{3}$.
10. $\frac{7}{8}-\frac{1}{6}$.
11. $\frac{14}{15}-\frac{1}{3}$.
12. $\frac{9}{16}-\frac{3}{8}$.
13. $\frac{11}{12}-\frac{6}{7}$.
14. $\frac{12}{17}-\frac{2}{3}$.
15. $\frac{11}{25}-\frac{1}{6}$.
16. $\frac{5}{21}-\frac{1}{7}$.
17. $\frac{7}{20}-\frac{2}{7}$.
18. $\frac{4}{13}-\frac{2}{9}$.
19. $\frac{8}{17}-\frac{12}{51}$.
20. $\frac{17}{25}-\frac{27}{50}$.
21. $\frac{31}{63}-\frac{8}{21}$.
22. $\frac{11}{30}-\frac{3}{10}$.
23. $\frac{19}{40}-\frac{7}{15}$.
24. $\frac{21}{35}-\frac{9}{28}$.
25. $\frac{27}{28}-\frac{9}{40}$.
26. $\frac{14}{41}-\frac{3}{13}$.
27. $25-18\frac{4}{5}$.
28. $37-15\frac{5}{6}$.
29. $42-12\frac{1}{6}$.
30. $56-10\frac{3}{5}$.
31. $71-20\frac{2}{3}$.
32. $90-31\frac{1}{2}$.
33. $64-29\frac{3}{7}$.
34. $47-40\frac{1}{8}$.

35. $39\frac{4}{15}-12$.
36. $28\frac{9}{16}-20$.
37. $71\frac{3}{17}-31$.
38. $48\frac{6}{19}-15$.
39. $82\frac{7}{18}-27$.
40. $56\frac{4}{13}-49$.
41. $42\frac{3}{4}-36\frac{4}{9}$.
42. $92\frac{4}{7}-25\frac{1}{2}$.
43. $75\frac{3}{8}-63\frac{1}{6}$.
44. $87\frac{1}{2}-39\frac{4}{5}$.
45. $51\frac{2}{3}-37\frac{1}{2}$.
46. $67\frac{8}{9}-24\frac{7}{8}$.
47. $200\frac{4}{57}-191\frac{8}{19}$.
48. $324\frac{1}{13}-180\frac{7}{40}$.
49. $502\frac{7}{15}-121\frac{16}{25}$
50. $2\frac{181}{435}-1\frac{145}{870}$.
51. $600\frac{201}{300}-437\frac{47}{666}$
52. $1001\frac{3}{77}-906\frac{4}{11}$.

MENTAL EXERCISES.

1. Joseph paid $\frac{9}{10}$ of a dollar for an Arithmetic, and $\frac{5}{6}$ of a dollar for a History. For which did he pay the more, and how much?

2. A man owning a factory, sold $\frac{9}{37}$ of it; how much did he still own?

3. Thomas gathered $9\frac{3}{4}$ quarts of strawberries, and sold $7\frac{8}{9}$ quarts; how many quarts had he left?

4. Henry will be 27 years old $12\frac{11}{15}$ years hence; how old is he now?

5. Having $\$39\frac{11}{20}$ in the bank, what amount must I add to it to make the sum \$60?

6. A grocer sold $\frac{1}{3}$ of a box of tea from a box $\frac{4}{5}$ full; what part of the whole box was left?

7. William earns $\$15\frac{9}{20}$ a week, and Louis $\$30\frac{13}{40}$; how much does one earn more than the other?

8. If I buy a slate for $6\frac{3}{4}$ cents, a Reader for $65\frac{2}{3}$ cents, and a Grammar for $27\frac{7}{12}$ cents, how much change will I receive from a \$5 bill?

9. Mr. Brown sold $\frac{1}{5}$ of his estate to A, $\frac{1}{5}$ to B, and $\frac{4}{15}$ to C; what part remained unsold?

10. If I had $\$8\frac{2}{3}$ more than I have, I should have $\$23\frac{1}{2}$. How much have I?

11. One man travels $37\frac{1}{4}$ miles a day, and another $31\frac{5}{8}$ miles a day. How many miles does one travel more than the other?

WRITTEN EXERCISES.

1. A man bought a farm, $236\frac{5}{8}$ acres, and sold $187\frac{9}{16}$ acres of it; how many acres had he remaining?

Ans. $49\frac{1}{16}$ acres.

2. Having \$$184\frac{9}{10}$, I bought a certain amount of cloth and had \$$63\frac{13}{15}$ left; what did the cloth cost me?

Ans. \$$121\frac{1}{30}$.

3. What number must be added to $164\frac{5}{9}$ to make $237\frac{7}{18}$?

Ans. $72\frac{5}{6}$.

4. Sold a barrel of flour for \$$9\frac{3}{4}$, and a barrel of potatoes for \$$4\frac{6}{7}$. Which cost the more, and how much?

Ans. Flour, \$$4\frac{25}{28}$.

5. From a cask containing $60\frac{3}{4}$ gallons of cider I drew off $17\frac{11}{12}$ gallons; how many gallons were left?

Ans. $42\frac{5}{6}$ gallons.

6. A has \$$16\frac{1}{2}$, B has \$$17\frac{2}{3}$, and C has \$$8\frac{4}{5}$; how much less than \$$78\frac{19}{30}$ have they all? *Ans.* \$$35\frac{2}{3}$.

7. A lady having \$150, made the following purchases: a bonnet for \$$10\frac{1}{2}$, a cloak for \$$47\frac{1}{6}$, and a shawl for \$$65\frac{1}{3}$. How much had she left? *Ans.* \$27.

8. A merchant has 2 casks of wine each containing $56\frac{5}{8}$ gallons. If he sell $33\frac{4}{7}$ gallons from each of them, how many gallons will he have left?

Ans. $46\frac{3}{28}$ gallons.

9. A father bequeathed \$5000 to his three sons. The first received \$$816\frac{5}{9}$; the second, \$$1684\frac{3}{5}$; and the third, the remainder. What did the third receive?

Ans. \$$2498\frac{38}{45}$.

10. Sold a quantity of grain, that cost me \$$6782\frac{3}{4}$, for \$$4896\frac{1}{2}$. I afterwards bought a second quantity for \$$6115\frac{11}{12}$, and sold it for \$8975. What did I gain or lose on both transactions? *Ans.* Gained \$$972\frac{5}{6}$.

In what time will:

2. $120 at 5%, give $54 int.? $400 at 7%, $840 int.?
3. $300 at 6%, give $36 int.? $320 at 6¼%, $400 int.?
4. $100 at 4%, give $40 int.? $500 at 7%, $350 int.?
5. $160 at 5%, give $56 int.? $600 at 5%, $756 int.?
6. $200 at 8%, give $64 int.? $700 at 3%, $847 int.?
7. $150 at 6%, give $90 int.? $300 at 4%, $721 int.?
8. $130 at 4%, give $52 int.? $100 at 9%, $183 int.?
9. $250 at 8%, give $100 int.? $200 at 3½%, $140 int.?
10. $140 at 5%, give $280 int.? $200 at 6%, $66 int.?
11. $175 at 8%, amount to $245? At 4%? At 2%?
12. $400 at 5%, amount to $520? At 2½%? At 10%?
13. $300 at 4%, amount to $396? At 8%? At 2%?
14. $100 at 7%, amount to $135? At 14%? At 21%?
15. $60 at 3%, amount to $69? At 6%? At 12%?
16. $60 at 8%, amount to $72? At 4%? At 16%?
17. $140 at 6%, amount to $182? At 1%? At 3%?
18. $30 at 5%, amount to $33? At 10%? At 20%?
19. $50 at 4%, amount to $62? At 8%? At 2%?
20. $200 at 6%, amount to $242? At 3%? At 12%?
21. $25 at 8%, amount to $35? At 4%? At 24%?
22. $160 at 5%, amount to $194? At 10%? At 20%?
23. $80 at 7%, amount to $108? At 14%? At 1%?
24. $300 at 4%, amount to $412? At 16%? At 24%?
25. $100 at 6%, amount to $124? At 3%? At 12%?
26. $500 at 10%, amount to $800? At 5%? At 30%?

27. In what time will any given principal double itself at 5%?

SOLUTION.—To double itself any sum of money must gain 100% of itself. At 1 per cent., it will require 100 years; and at 5% it will require as many years as 5% is contained times in 100%, which are 20 times. Therefore, it will require 20 years for any principal to double itself at 5%.

MULTIPLICATION.

101. ***Multiplication of Fractions*** is the process of finding the product of two or more fractions.

CASE I.

102. To multiply a fraction by a whole number.

ORAL EXERCISES.

1. Multiply $\frac{5}{8}$ by 4. $3\frac{1}{2} \times 7$.

SOLUTION.—To multiply a fraction by an integer, we may either multiply the numerator, or divide the denominator by the integer. Thus, $\frac{5}{8} \times 4 = \frac{20}{8} = 2\frac{1}{2}$, *Ans.* Or, $\frac{5}{8} \times 4 = \frac{5}{2} = 2\frac{1}{2}$, *Ans.*

In multiplying mixed numbers, we may reduce them to improper fractions, or multiply their parts separately, and add the results. Thus, $3\frac{1}{2} \times 7 = \frac{7}{2} \times 7 = \frac{49}{2} = 24\frac{1}{2}$, *Ans.* Or, $3\frac{1}{2} \times 7 = (3 \times 7) + (\frac{1}{2} \times 7) = 21 + 3\frac{1}{2} = 24\frac{1}{2}$, *Ans.*

I.

2. $\frac{1}{9} \times 4$.
3. $\frac{3}{8} \times 2$.
4. $\frac{5}{6} \times 3$.
5. $\frac{4}{7} \times 5$.
6. $\frac{4}{5} \times 6$.
7. $\frac{3}{7} \times 4$.
8. $\frac{2}{9} \times 3$.
9. $\frac{3}{4} \times 2$.
10. $\frac{7}{9} \times 3$.
11. $\frac{2}{5} \times 5$.
12. $\frac{7}{8} \times 6$.
13. $\frac{8}{9} \times 7$.
14. $\frac{3}{4} \times 4$.
15. $\frac{5}{6} \times 5$.
16. $\frac{6}{7} \times 3$.
17. $\frac{3}{5} \times 8$.
18. $\frac{7}{15} \times 2$.
19. $\frac{7}{18} \times 3$.
20. $\frac{2}{17} \times 7$.
21. $\frac{5}{21} \times 7$.
22. $\frac{8}{11} \times 9$.
23. $\frac{3}{24} \times 4$.
24. $\frac{3}{22} \times 5$.
25. $\frac{3}{13} \times 6$.
26. $2\frac{1}{2} \times 4$.
27. $3\frac{3}{4} \times 5$.
28. $7\frac{3}{5} \times 4$.
29. $8\frac{1}{6} \times 8$.
30. $6\frac{3}{4} \times 5$.
31. $9\frac{2}{7} \times 6$.
32. $3\frac{5}{8} \times 4$.
33. $8\frac{3}{4} \times 7$.

II.

1. At \$$\frac{3}{4}$ apiece, what will five penknives cost?
2. At \$$\frac{3}{8}$ a yard, what will 7 yards of alpaca cost?
3. What will 6 base balls cost at \$$\frac{2}{3}$ each?
4. What will 8 pounds of cheese cost \$$\frac{1}{7}$ a pound?

5. Find the value of 16 barrels of flour at \$$8\frac{1}{2}$ per barrel.

6. If I gain \$$\frac{5}{8}$ on one yard of cloth, how much will I gain on 15 yards?

7. What will eleven bushels of apples cost at \$$\frac{5}{6}$ a bushel? 18 bushels? 30 bushels?

8. If a pound of tea cost \$$\frac{3}{8}$, what will 16 pounds cost?

9. What is the cost of 3 barrels of flour at \$$8\frac{1}{3}$ a barrel? \$$7\frac{1}{2}$? \$$9\frac{1}{4}$? \$$6\frac{3}{4}$?

10. What is the cost of 6 tons of coal at \$$6\frac{1}{4}$ a ton?

11. What amount of money will pay for 5 weeks' board at \$$7\frac{2}{3}$ a week?

12. What is the product of $2\frac{1}{2} + 3\frac{3}{4}$, multiplied by 8?

WRITTEN EXERCISES.

1. Multiply $\frac{4}{9}$ by 3.

OPERATION.

$\frac{4}{9} \times 3 = \frac{12}{9} = 1\frac{1}{3}$, *Ans.*

Or,

$\frac{4}{9} \times 3 = \frac{4}{3} = 1\frac{1}{3}$, *Ans.*

SOLUTION.—To multiply the fraction $\frac{4}{9}$ by 3, we multiply the numerator 4 by 3, and keep the same denominator. This operation gives $\frac{12}{9} = 1\frac{1}{3}$, for the required result.

Or, since 3, the multiplier, is a factor of 9, the denominator of the multiplicand, we may also perform the multiplication by dividing the denominator by the multiplier. Thus we obtain $\frac{4}{3} = 1\frac{1}{3}$, *Ans.*

What is the value of:

2. $\frac{2}{3} \times 12$.
3. $\frac{3}{4} \times 20$.
4. $\frac{8}{9} \times 36$.
5. $\frac{7}{8} \times 40$.
6. $\frac{6}{7} \times 32$.
7. $\frac{5}{8} \times 17$.
8. $\frac{4}{9} \times 54$.
9. $\frac{2}{9} \times 45$.
10. $\frac{19}{21} \times 7$.
11. $\frac{16}{51} \times 3$.
12. $\frac{9}{37} \times 6$.
13. $\frac{11}{15} \times 5$.
14. $\frac{8}{65} \times 7$.
15. $\frac{7}{17} \times 9$.
16. $\frac{13}{85} \times 5$.
17. $\frac{23}{28} \times 4$.
18. $131\frac{4}{5} \times 9$.
19. $215\frac{5}{9} \times 6$.
20. $182\frac{5}{6} \times 4$.
21. $360\frac{2}{3} \times 5$.
22. $542\frac{3}{4} \times 8$.
23. $418\frac{1}{2} \times 7$.
24. $204\frac{5}{8} \times 9$.
25. $605\frac{1}{6} \times 6$.
26. $\frac{57}{132} \times 66$.
27. $\frac{102}{325} \times 25$.
28. $19\frac{99}{100} \times 20$.
29. $121\frac{11}{24} \times 36$.
30. $518\frac{9}{10} \times 45$.
31. $363\frac{8}{15} \times 56$.
32. $284\frac{7}{12} \times 48$.
33. $110\frac{19}{30} \times 50$.

CASE II.

103. To multiply a whole number by a fraction.

ORAL EXERCISES.

1. Multiply 6 by $\frac{2}{3}$.

SOLUTION.—Multiply the whole number by the numerator and divide the product by the denominator. Thus, $6 \times \frac{2}{3} = \frac{12}{3} = 4$, *Ans.* Or, $6 \times \frac{2}{3} = 2 \times \frac{1}{3}$ of $6 = 4$, *Ans.* When mixed numbers occur, perform the operation as in Case I.

I.

2. $3 \times \frac{1}{4}$.
3. $6 \times \frac{2}{3}$.
4. $9 \times \frac{3}{4}$.
5. $7 \times \frac{1}{2}$.
6. $9 \times \frac{3}{5}$.
7. $3 \times \frac{4}{7}$.
8. $8 \times \frac{5}{6}$.
9. $5 \times \frac{2}{9}$.
10. $7 \times \frac{3}{7}$.
11. $6 \times \frac{7}{8}$.
12. $3 \times \frac{5}{9}$.
13. $4 \times \frac{8}{9}$.
14. $9 \times \frac{4}{5}$.
15. $7 \times \frac{3}{2}$.
16. $8 \times \frac{9}{4}$.
17. $6 \times \frac{8}{3}$.
18. $5 \times \frac{8}{15}$.
19. $6 \times \frac{7}{54}$.
20. $4 \times \frac{17}{24}$.
21. $8 \times \frac{11}{16}$.
22. $9 \times \frac{14}{15}$.
23. $7 \times \frac{8}{21}$.
24. $6 \times \frac{11}{36}$.
25. $3 \times \frac{14}{39}$.
26. $9 \times 3\frac{1}{9}$.
27. $8 \times 5\frac{1}{6}$.
28. $7 \times 8\frac{3}{7}$.
29. $5 \times 3\frac{4}{5}$.
30. $3 \times 7\frac{5}{6}$.
31. $4 \times 8\frac{8}{9}$.
32. $9 \times 5\frac{2}{3}$.
33. $8 \times 7\frac{3}{4}$.

II.

1. Joseph having 3 cents' worth of candy, gave John $\frac{1}{3}$ of it. How much did John receive?

2. What will $\frac{6}{15}$ of a spool of thread cost at 5 cents a spool? $\frac{4}{5}$ of a spool? $\frac{9}{10}$ of a spool?

3. What is the $\frac{6}{63}$ of \$9? Of \$36? Of \$27? Of \$54? Of \$72? Of \$21? Of \$48?

4. At \$8 a week, what will a man earn in $\frac{1}{2}$ of a week?

5. When sugar is worth 13 cents a pound, what will $\frac{7}{26}$ of a pound cost? $\frac{8}{19}$ of a pound? $\frac{2}{3}$? $\frac{5}{6}$?

6. At 30 days to a month, how many days in $\frac{4}{15}$ of a month? In $\frac{5}{6}$? In $\frac{9}{10}$? In $\frac{2}{3}$?

7. At \$6 a yard, what will $\frac{7}{10}$ of a yard of cloth cost?

WRITTEN EXERCISES.

1. Multiply 8 by $\frac{3}{4}$.

OPERATION.

$$8 \times \frac{3}{4} = (8 \div 4) \times 3 = 6, \textit{ Ans.}$$

Or,

$$8 \times \frac{3}{4} = \frac{\overset{2}{\cancel{8}} \times 3}{\cancel{4}} = 6, \textit{ Ans.}$$

SOLUTION.—To multiply 8 by $\frac{3}{4}$ is the same as to find $\frac{3}{4}$ of 8. But $\frac{3}{4}$ of 8 is equal to 3 times $\frac{1}{4}$ of 8, which is 6.

Or, multiplying the whole number 8, by the numerator 3, and dividing by the denominator 4, we obtain 6 for result.

2. Multiply 25 by $4\frac{3}{5}$.

OPERATION.

$$\begin{array}{rl} 25 & \\ 4\frac{3}{5} & \\ \hline 15 & = \frac{3}{5} \text{ of } 25 \\ 100 & = 4 \text{ times } 25 \\ \hline 115 & = 4\frac{3}{5} \text{ times } 25 \end{array}$$

Or,

$$25 \times 4\frac{3}{5} = 25 \times \frac{23}{5} = 115, \textit{ Ans.}$$

SOLUTION.—To multiply 25 by $4\frac{3}{5}$ is the same as to add the $\frac{3}{5}$ of 25 to 4 times 25. Now, $\frac{3}{5}$ of 25 is equal to 15; and 4 times 25 is equal to 100. Adding 15 and 100 we get 115. Or, before multiplying we may reduce the mixed number, $4\frac{3}{5}$, to an improper fraction, and then perform the operation as in the preceding example.

Find the product of :

3. $35 \times \frac{5}{7}$.
4. $75 \times \frac{3}{5}$.
5. $81 \times \frac{8}{9}$.
6. $78 \times \frac{2}{3}$.
7. $67 \times \frac{3}{4}$.
8. $44 \times \frac{5}{6}$.
9. $58 \times \frac{4}{7}$.
10. $95 \times \frac{3}{8}$.
11. $120 \times \frac{7}{10}$.
12. $352 \times \frac{11}{12}$.
13. $460 \times \frac{7}{20}$.
14. $500 \times \frac{8}{15}$.
15. $148 \times \frac{3}{16}$.
16. $256 \times \frac{8}{13}$.
17. $692 \times \frac{4}{11}$.
18. $428 \times \frac{5}{14}$.
19. $396 \times 4\frac{1}{2}$.
20. $463 \times 6\frac{2}{3}$.
21. $850 \times 3\frac{4}{5}$.
22. $580 \times 7\frac{3}{4}$.
23. $325 \times 8\frac{5}{8}$.
24. $630 \times 7\frac{2}{9}$.
25. $245 \times 9\frac{3}{7}$.
26. $199 \times 3\frac{1}{8}$.

27. $66 \times 19\frac{5}{6}$.
28. $18 \times 50\frac{5}{9}$.
29. $61 \times 12\frac{1}{2}$.
30. $47 \times 41\frac{1}{3}$.
31. $89 \times 12\frac{2}{3}$.
32. $91 \times 16\frac{4}{7}$.
33. $625 \times 9\frac{1}{25}$.
34. $420 \times 2\frac{13}{21}$.
35. $760 \times 7\frac{9}{10}$.
36. $840 \times 6\frac{9}{14}$.
37. $101 \times 3\frac{2}{11}$.
38. $238 \times 5\frac{9}{13}$.
39. $316 \times 22\frac{1}{4}$.
40. $659 \times 14\frac{1}{7}$.
41. $285 \times 50\frac{5}{9}$.
42. $245 \times 25\frac{8}{35}$.
43. $461 \times 32\frac{9}{70}$.
44. $576 \times 40\frac{17}{36}$.

CASE III.

104. To find the product of two or more fractions or mixed numbers.

ORAL EXERCISES.

1. Multiply $\frac{2}{3}$ by $\frac{4}{5}$.

SOLUTION.—Multiply the numerators together for a new numerator, and the denominators for a new denominator. Thus, $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$, *Ans.* Reduce mixed numbers to improper fractions.

I.

2. $\frac{1}{2} \times \frac{1}{2}$.
3. $\frac{1}{2} \times \frac{1}{4}$.
4. $\frac{1}{3} \times \frac{2}{3}$.
5. $\frac{3}{4} \times \frac{2}{3}$.
6. $\frac{2}{7} \times \frac{3}{5}$.
7. $\frac{1}{3} \times \frac{3}{4}$.
8. $\frac{1}{4} \times \frac{1}{8}$.
9. $\frac{5}{6} \times \frac{1}{7}$.
10. $\frac{3}{4} \times \frac{9}{10}$.
11. $\frac{5}{7} \times \frac{14}{15}$.
12. $\frac{3}{5} \times \frac{10}{21}$.
13. $\frac{4}{7} \times \frac{21}{32}$.
14. $\frac{2}{5} \times \frac{11}{16}$.
15. $\frac{5}{6} \times \frac{12}{5}$.
16. $\frac{8}{5} \times \frac{7}{11}$.
17. $\frac{7}{9} \times \frac{9}{10}$.
18. $\frac{8}{15} \times \frac{10}{40}$.
19. $\frac{9}{14} \times \frac{11}{4}$.
20. $\frac{17}{18} \times \frac{9}{20}$.
21. $\frac{24}{25} \times \frac{15}{16}$.
22. $\frac{7}{30} \times \frac{10}{13}$.
23. $\frac{8}{45} \times \frac{15}{9}$.
24. $\frac{8}{13} \times \frac{39}{40}$.
25. $\frac{7}{10} \times \frac{12}{35}$.
26. $\frac{3}{4} \times 1\frac{1}{2}$.
27. $\frac{5}{8} \times 2\frac{2}{5}$.
28. $\frac{1}{2} \times 9\frac{1}{2}$.
29. $\frac{4}{7} \times 3\frac{1}{6}$.
30. $\frac{4}{5} \times 5\frac{1}{4}$.
31. $\frac{2}{3} \times 1\frac{5}{7}$.
32. $\frac{3}{5} \times 6\frac{2}{3}$.
33. $\frac{8}{9} \times 2\frac{3}{4}$.

II.

1. If a ton of coal cost $\$5\frac{1}{2}$, what will $\frac{2}{7}$ of a ton cost?

2. What cost $\frac{3}{4}$ of a yard of muslin at $12\frac{1}{2}$ cents a yard?

3. What will be the cost of $\frac{3}{5}$ of a quart of chestnuts at $7\frac{1}{3}$ cents a quart?

4. What is the $\frac{1}{7}$ of $14\frac{7}{10}$? $\frac{3}{8}$ of $2\frac{1}{2}$? $\frac{6}{7}$ of $9\frac{1}{3}$?

5. James having \$$\frac{2}{3}$ gave his brother $\frac{1}{5}$ of it; what part of a dollar did his brother receive?

6. When rye is worth \$$\frac{5}{6}$ a bushel, what will $\frac{4}{5}$ of a bushel cost?

7. At \$$2\frac{1}{2}$ a yard, what will $\frac{3}{4}$ of a yard of cloth cost?

8. What will $\frac{1}{2}$ a pound of tea cost at \$$\frac{2}{3}$ a pound?

WRITTEN EXERCISES.

1. Multiply $\frac{5}{9}$ by $\frac{3}{7}$.

OPERATION.

$$\frac{5}{9}\times\frac{3}{7}=\frac{5}{9}\times3=\frac{15}{9}$$

$$\frac{15}{9}\div7=\frac{15}{9\times7}=\frac{15}{63}=\frac{5}{21},\ \textit{Ans.}$$

Or,

$$\frac{5}{9}\times\frac{3}{7}=\frac{5\times\cancel{3}}{\underset{3}{\cancel{9}}\times7}=\frac{5}{21},\ \textit{Ans.}$$

Solution.—To multiply $\frac{5}{9}$ by $\frac{3}{7}$ we first multiply $\frac{5}{9}$ by 3, which gives $\frac{15}{9}$. But this result is 7 times the required product, because 3 is 7 times the given multiplier. Hence, to find the true product we must divide the product of $\frac{5}{9}\times3$ by 7. This gives $\frac{5\times\cancel{3}}{\cancel{9}\times7}=\frac{5}{21}$, *Ans.*

Note.—Cancel all factors common to the numerators and denominators.

2. Multiply $64\frac{1}{2}$ by $48\frac{2}{3}$.

OPERATION.

$$\begin{array}{rrcl}
 & 64\frac{1}{2} & & \\
 & \underline{48\frac{2}{3}} & & \\
 & \frac{1}{3} & = & \frac{1}{2}\times\frac{2}{3} \\
3\ )\ 128 = & 42\frac{2}{3} & = & 64\times\frac{2}{3} \\
2\ )\ 48 = & 24 & = & \frac{1}{2}\times48 \\
 & \left.\begin{array}{r}512\\ 256\ \ \end{array}\right\} & = & 64\times48 \\
 & \overline{3139} & = & 64\frac{1}{2}\times48\frac{2}{3}
\end{array}$$

Solution. — Instead of reducing to improper fractions, as when the numbers are small, multiply the whole number and the fraction of the multiplicand separately, first by the fraction and then by the whole of the multiplier. Take the sum of the partial products for the required product.

Find the product of:

3. $\frac{5}{12}\times\frac{2}{3}$
4. $\frac{13}{21}\times\frac{1}{2}$
5. $\frac{22}{35}\times\frac{7}{9}$
6. $\frac{14}{15}\times\frac{5}{8}$
7. $\frac{19}{25}\times\frac{5}{6}$
8. $\frac{16}{31}\times\frac{1}{8}$
9. $\frac{7}{23}\times\frac{3}{7}$
10. $\frac{7}{36}\times\frac{4}{5}$

11. $\frac{3}{5}\times\frac{5}{8}\times\frac{6}{7}$
12. $\frac{1}{2}\times\frac{7}{9}\times\frac{3}{4}$
13. $\frac{6}{7}\times\frac{7}{8}\times\frac{2}{3}$
14. $\frac{1}{4}\times\frac{2}{3}\times\frac{8}{9}$
15. $\frac{3}{8}\times\frac{4}{7}\times\frac{1}{3}$
16. $\frac{4}{5}\times\frac{1}{8}\times\frac{5}{7}$
17. $\frac{8}{9}\times\frac{1}{2}\times\frac{1}{7}$
18. $\frac{9}{7}\times\frac{3}{2}\times\frac{4}{7}$

19. $\frac{5}{8}\times12\frac{1}{2}$
20. $\frac{8}{9}\times22\frac{1}{2}$
21. $\frac{2}{3}\times35\frac{1}{4}$
22. $\frac{6}{7}\times76\frac{3}{5}$
23. $\frac{5}{8}\times29\frac{1}{3}$
24. $\frac{1}{7}\times84\frac{2}{5}$
25. $\frac{4}{5}\times12\frac{4}{5}$
26. $\frac{3}{4}\times62\frac{1}{2}$

27. $41\frac{2}{3}\times12\frac{1}{2}$.
28. $15\frac{1}{8}\times10\frac{1}{3}$.
29. $62\frac{1}{2}\times15\frac{1}{4}$.
30. $27\frac{1}{3}\times42\frac{3}{4}$.
31. $78\frac{3}{4}\times26\frac{4}{5}$.
32. $34\frac{2}{7}\times15\frac{7}{9}$.
33. $48\frac{4}{5}\times30\frac{1}{4}$.
34. $52\frac{6}{7}\times65\frac{1}{2}$.

35. $230\frac{3}{4}\times\frac{1}{2}$.
36. $189\frac{2}{3}\times\frac{2}{5}$.
37. $350\frac{1}{4}\times\frac{2}{3}$.
38. $436\frac{2}{5}\times\frac{5}{8}$.
39. $170\frac{2}{9}\times\frac{3}{4}$.
40. $520\frac{3}{7}\times\frac{1}{3}$.
41. $280\frac{1}{4}\times\frac{8}{9}$.
42. $600\frac{1}{2}\times\frac{4}{5}$.

43. $360\frac{1}{4}\times50\frac{1}{3}$.
44. $135\frac{1}{2}\times42\frac{1}{4}$.
45. $270\frac{2}{3}\times32\frac{1}{5}$.
46. $307\frac{1}{4}\times18\frac{1}{2}$.
47. $405\frac{2}{5}\times32\frac{6}{7}$.
48. $154\frac{1}{6}\times21\frac{1}{3}$.
49. $193\frac{2}{3}\times24\frac{1}{8}$.
50. $200\frac{3}{4}\times27\frac{2}{5}$.

51. $221\frac{1}{3}\times20\frac{3}{4}$.
52. $430\frac{1}{2}\times16\frac{4}{7}$.
53. $387\frac{2}{3}\times47\frac{1}{2}$.
54. $564\frac{7}{9}\times32\frac{1}{7}$.
55. $102\frac{1}{4}\times56\frac{1}{3}$.
56. $670\frac{2}{5}\times10\frac{5}{12}$.

57. $\frac{6}{17}\times\frac{34}{120}\times5\frac{5}{8}$.
58. $16\frac{2}{3}\times17\frac{8}{9}\times\frac{27}{50}$.
59. $48\times\frac{2}{7}\times3\frac{2}{11}\times\frac{11}{15}$.
60. $(\frac{2}{3}+\frac{4}{7})\times(\frac{2}{7}-\frac{3}{14})$.
61. $(2\frac{3}{5}-1\frac{1}{2})\times(16\frac{1}{4}\times8\frac{2}{5})\times1\frac{3}{17}$.
62. $[(140-19\frac{2}{3})\times(52\frac{1}{2}+3\frac{1}{4})]\times21\frac{3}{5}$.

MENTAL EXERCISES.

1. **What will 4 barrels of flour cost at $\$7\frac{5}{8}$ a barrel?**

2. **How many bushels of wheat will 27 bags hold, if each hold $3\frac{1}{3}$ bushels?**

3. **If it take $4\frac{1}{2}$ yards of cloth for a coat, and $3\frac{1}{8}$ yards for a pair of pantaloons, how many yards will be required to make 4 of each?**

4. How long will 1 man take to do a certain piece of work, if 8 men require $12\frac{1}{2}$ days to do it?

5. If a person's expenses be $15 a week, what will he spend in $7\frac{3}{5}$ weeks? $8\frac{2}{3}$ weeks?

6. Joseph having $\frac{3}{4}$ of a pound of candy, gave $\frac{1}{3}$ of it to James, $\frac{1}{6}$ to William, and $\frac{1}{4}$ to Henry. What part of a pound had he left?

7. Mr. Smith owning $\frac{2}{5}$ of an iron foundry, sold $\frac{1}{6}$ of his share. What part had he left?

8. A man lost $\frac{1}{3}$ of his money at play, and then won $\frac{1}{4}$ as much as he had lost. What part of his money had he then?

9. At \$$3\frac{1}{6}$ a yard what will $4\frac{4}{5}$ yards of cloth cost?

10. Bought a wagon for \$$35\frac{7}{8}$, and sold it for $\frac{4}{7}$ of what it cost. What was the loss?

11. What will $\frac{1}{8}$ of $\frac{1}{9}$ of 216 gallons of oil cost, at \$$\frac{7}{8}$ a gallon? At \$$\frac{3}{4}$? At \$$\frac{5}{6}$?

12. A man paid \$$15\frac{2}{3}$ for an overcoat, and $\frac{1}{2}$ as much for a vest. How much did he pay for both?

13. Thomas gave each of 6 poor persons $\frac{3}{7}$ of a dollar, and had $5\frac{1}{4}$ dollars left; how much had he at first?

14. If 6 pears cost $\frac{7}{8}$ of a dime apiece, what will 24 cost? 16? 32? 12? 14?

15. James having spent $\frac{5}{9}$ of his money, found as much as $\frac{1}{4}$ of the remainder. What part of his money did he then have?

16. An orchard contains 56 trees, $\frac{3}{8}$ of which are pear trees; $\frac{2}{7}$ of the remainder are peach trees; and the rest bear apples. How many apple trees in the orchard?

17. If one dozen of eggs cost \$$\frac{7}{20}$, what will 12 dozen cost? 25 dozen? 44 dozen?

18. Find the cost of 20 pounds of cheese at $11\frac{3}{4}$ cents a pound. 30 pounds. 16 pounds.

WRITTEN EXERCISES.

1. A farmer bought 27 acres of land at $\$96\frac{3}{4}$ an acre, and 75 head of cattle at $\$23\frac{1}{2}$ per head. What did he pay for all? *Ans.* $\$4374\frac{3}{4}$.

2. A flour merchant bought 565 barrels of flour at $\$5\frac{2}{3}$ per barrel. He sold 420 barrels at $\$3\frac{1}{4}$ a barrel, and the remainder at $\$8\frac{2}{5}$. How much did he lose? *Ans.* $\$618\frac{2}{3}$.

3. What will be the cost of $\frac{4}{9}$ of a piece of cloth containing $337\frac{1}{2}$ yards, at $\$5\frac{1}{4}$ a yard? *Ans.* $\$787\frac{1}{2}$.

4. A has $\$371\frac{7}{8}$; B has $16\frac{2}{3}$ times as much; and C has $\$270\frac{1}{4}$ more than A and B together. How much have they all? *Ans.* $\$13409\frac{5}{6}$.

5. A daughter is $19\frac{5}{12}$ years old, and her mother lacks $9\frac{6}{7}$ years of being 3 times as old. What are their united ages? *Ans.* $67\frac{17}{21}$ years.

6. A man travelled $218\frac{1}{2}$ miles in three days. The first day he travelled $\frac{1}{5}$ of the whole distance, and the second, $\frac{1}{3}$ as far as he travelled on the first day. How far did he travel on the third day? *Ans.* $160\frac{7}{30}$ miles.

7. If a train of cars run $25\frac{1}{4}$ miles in an hour, how far will it run in $6\frac{3}{7}$ hours? *Ans.* $162\frac{9}{28}$ miles.

8. Paid $\$249\frac{5}{8}$ for a horse and sold him for $\frac{2}{3}$ of what he cost. What was the loss? *Ans.* $\$83\frac{5}{24}$.

9. When raisins are worth $\$\frac{9}{10}$ a box, what will $135\frac{4}{5}$ boxes cost? *Ans.* $\$122\frac{11}{50}$.

10. Two sevenths of a pole is in the mud, $\frac{1}{6}$ of it is in the water, and the rest of it is in the air. How much less than $\frac{3}{4}$ of it is in the air? *Ans.* $\frac{17}{84}$ feet.

11. One person spent \$2 for coal worth \$5 a ton; and another \$4 for a quantity worth \$6 per ton. How much does one purchase more than the other? *Ans.* $\frac{4}{15}$ ton.

12. From a flock of sheep containing 700, I lost $\frac{5}{14}$, and afterward added $\frac{2}{9}$ as many as remained. By what part of itself was the original flock increased or diminished?

Ans. Diminished $\frac{3}{14}$.

13. A gentleman having $\frac{7}{8}$ of $213\frac{5}{6}$ acres of land, sold $\frac{3}{7}$ of his share. How many acres did he sell?

Ans. $80\frac{3}{16}$ acres.

14. What will be the cost of $\frac{3}{5}$ of $\frac{9}{10}$ of a ton of hay at $\$4\frac{4}{5}$ per ton? *Ans.* $\$2\frac{74}{125}$.

15. A boy sold $9\frac{3}{4}$ dozen of eggs at 3 cents apiece. He received in payment $6\frac{5}{8}$ pounds of butter at 20 cents a pound, and $2\frac{1}{3}$ yards of ribbon at 4 cents a yard; how much was still due him? *Ans.* $\$2.09\frac{1}{6}$.

16. Mr. Carson rented a farm at $\$54\frac{1}{4}$ a month, taking a lease for six years, but disposed of the lease at the end of $4\frac{2}{3}$ years. How much rent did he pay? *Ans.* $3038.

17. A man at his death left his wife $18400, which was $\frac{1}{3}$ of his estate. She at her death left $\frac{3}{8}$ of her share to her daughter. What part of the father's estate did the daughter receive? *Ans.* $\frac{1}{8}$.

18. If a horse eat $\frac{2}{3}$ of a bushel of oats in a day, how many bushels will 20 horses eat in $8\frac{1}{2}$ weeks?

Ans. $793\frac{1}{3}$ bushels.

19. At the rate of 3640 words in an hour, how many words will a man utter in $\frac{17}{20}$ of an hour?

Ans. 3094 words.

20. If by working 13 hours a day, a piece of work can be done in $53\frac{2}{3}$ days, in what time can it be done by working one hour a day? *Ans.* $697\frac{2}{3}$ days.

21. Find the cost of the following: 27 barrels of apples at $\$3\frac{2}{3}$ a barrel, 39 bushels of strawberries at $\$4\frac{1}{2}$ a bushel, and 140 quarts of nuts at $14\frac{1}{2}$ cents a quart.

DIVISION.

105. ***Division of Fractions*** is the process of dividing when one or both of the terms are fractional.

CASE I.

106. To divide a fraction by an integer.

ORAL EXERCISES.

1. Divide $\frac{4}{5}$ by 2.

SOLUTION.—Divide the numerator by the whole number, and keep the same denominator; or multiply the denominator by the whole number, and keep the same numerator. Thus, $\frac{4}{5} \div 2 = \frac{2}{5}$, *Ans.* Or, $\frac{4}{5} \div 2 = \frac{4}{10} = \frac{2}{5}$, *Ans.*

I.

2. $\frac{4}{5} \div 2$.
3. $\frac{2}{3} \div 2$.
4. $\frac{3}{4} \div 4$.
5. $\frac{1}{2} \div 3$.
6. $\frac{2}{5} \div 6$.
7. $\frac{8}{9} \div 4$.
8. $\frac{7}{8} \div 3$.
9. $\frac{5}{6} \div 5$.
10. $\frac{3}{7} \div 6$.
11. $\frac{5}{7} \div 4$.
12. $\frac{8}{9} \div 9$.
13. $\frac{5}{8} \div 3$.
14. $\frac{4}{5} \div 8$.
15. $\frac{7}{9} \div 6$.
16. $\frac{7}{8} \div 5$.
17. $\frac{5}{4} \div 8$.
18. $\frac{10}{11} \div 5$.
19. $\frac{9}{10} \div 4$.
20. $\frac{14}{15} \div 3$.
21. $\frac{6}{13} \div 2$.
22. $\frac{12}{17} \div 6$.
23. $\frac{8}{25} \div 4$.
24. $\frac{15}{2} \div 7$.
25. $\frac{42}{9} \div 5$.
26. $3\frac{3}{4} \div 3$.
27. $5\frac{1}{2} \div 6$.
28. $8\frac{3}{4} \div 7$.
29. $6\frac{7}{8} \div 2$.
30. $4\frac{4}{5} \div 8$.
31. $9\frac{1}{2} \div 5$.
32. $7\frac{3}{5} \div 2$.
33. $1\frac{7}{8} \div 5$.

II.

1. James shared $\frac{3}{4}$ of a dollar equally between two companions; how much did he give each?

2. Henry paid $\frac{8}{9}$ of a dollar for 3 bats; what was each worth?

3. Mary did $\frac{3}{5}$ of her work in 4 days; what part of the work did she do in a day?

4. If 10 rubber balls cost $\frac{7}{8}$ of a dollar, what will 1 rubber ball cost?

5. Divide $\frac{6}{7}$ of a ton of coal among 3 poor families; what part of a ton will each receive?

6. Divide $\frac{5}{6}$ of a dollar into 5 equal parts.

7. How many times are 3 contained in $\frac{7}{8}$?

WRITTEN EXERCISES.

1. Divide $\frac{6}{13}$ by 3.

OPERATION.

$$\tfrac{6}{13} \div 3 = \frac{6 \div 3}{13} = \tfrac{2}{13}$$

Or,

$$\tfrac{6}{13} \div 3 = \frac{6}{13 \times 3} = \tfrac{2}{13}$$

Solution.—To divide $\frac{6}{13}$ by 3 we may either divide the numerator or multiply the denominator by 3; which in each case gives $\frac{2}{13}$ for result. By the first operation we diminish the *number* of parts; and by the second, we diminish their *value*. In both cases the result is the same.

Notes.—1. Whenever the numerator is a multiple of the divisor, the operation should be performed by dividing the numerator.

2. Mixed numbers should be reduced to improper fractions before dividing.

Find the value of:

2. $\frac{10}{17} \div 4$.
3. $\frac{20}{33} \div 5$.
4. $\frac{14}{27} \div 4$.
5. $\frac{16}{21} \div 8$.
6. $\frac{3}{11} \div 7$.
7. $\frac{15}{28} \div 9$.
8. $\frac{9}{17} \div 6$.
9. $\frac{18}{25} \div 8$.
10. $\frac{36}{47} \div 12$.
11. $\frac{28}{30} \div 14$.
12. $\frac{45}{82} \div 15$.
13. $\frac{11}{12} \div 16$.
14. $\frac{81}{93} \div 27$.
15. $\frac{135}{171} \div 15$.
16. $\frac{128}{204} \div 16$.
17. $\frac{225}{316} \div 25$.
18. $10\frac{3}{5} \div 8$.
19. $25\frac{1}{2} \div 6$.
20. $37\frac{1}{2} \div 5$.
21. $16\frac{1}{3} \div 7$.
22. $15\frac{1}{8} \div 9$.
23. $10\frac{1}{4} \div 6$.
24. $12\frac{1}{3} \div 5$.
25. $75\frac{3}{5} \div 4$.

26. $125\frac{1}{4} \div 12$.
27. $250\frac{2}{3} \div 20$.
28. $327\frac{1}{5} \div 15$.
29. $400\frac{3}{4} \div 24$.
30. $156\frac{1}{2} \div 33$.
31. $194\frac{1}{8} \div 16$.
32. $260\frac{1}{9} \div 12$.
33. $345\frac{1}{3} \div 16$.
34. $135\frac{7}{8} \div 25$.
35. $440\frac{2}{7} \div 30$.
36. $137\frac{1}{2} \div 50$.
37. $210\frac{4}{5} \div 60$.

CASE II.

107. To divide an integer by a fraction.

ORAL EXERCISES.

1. Divide 5 by $\frac{3}{4}$.

SOLUTION.—Multiply the whole number by the denominator of the fraction, and divide the product by the numerator. Thus, $5 \div \frac{3}{4} = \frac{20}{3} = 6\frac{2}{3}$, *Ans.*

I.

2 $6 \div \frac{2}{3}$.
3. $9 \div \frac{3}{4}$.
4. $6 \div \frac{4}{5}$.
5. $8 \div \frac{1}{5}$.
6. $5 \div \frac{3}{7}$.
7. $8 \div \frac{6}{7}$.
8. $7 \div \frac{2}{3}$.
9. $9 \div \frac{1}{4}$.
10. $12 \div \frac{4}{5}$.
11. $15 \div \frac{5}{6}$.
12. $16 \div \frac{7}{8}$.
13. $27 \div \frac{9}{2}$.
14. $12 \div \frac{3}{7}$.
15. $18 \div \frac{3}{5}$.
16. $32 \div \frac{3}{4}$.
17. $15 \div \frac{6}{7}$.
18. $16 \div \frac{4}{5}$.
19. $10 \div \frac{5}{8}$.
20. $20 \div \frac{2}{3}$.
21. $14 \div \frac{7}{8}$.
22. $25 \div \frac{5}{3}$.
23. $30 \div \frac{3}{4}$.
24. $18 \div \frac{6}{7}$.
25. $24 \div \frac{8}{9}$.
26. $39 \div 3\frac{1}{4}$.
27. $40 \div 1\frac{3}{5}$.
28. $36 \div 1\frac{1}{2}$.
29. $49 \div 6\frac{1}{8}$.
30. $25 \div 8\frac{1}{3}$.
31. $42 \div 8\frac{2}{5}$.
32. $18 \div 4\frac{1}{2}$.
33. $32 \div 1\frac{7}{9}$.

II.

1. How many packages of tea, containing $\frac{2}{7}$ of a pound each, can be made from 8 pounds?

2. 18 pounds are $\frac{3}{4}$ of how many pounds?

3. If $\frac{3}{7}$ of a piece of work can be done in 6 days, what time will it take to do $\frac{1}{7}$ of it?

4. How many fourths in 6 pounds? In 8 pounds?

5. If a boy earn $\frac{5}{8}$ of a dollar a day, in what time will he earn \$8? \$10? \$5?

6. At $\frac{2}{5}$ of a dollar a yard, how many yards can be bought for \$10?

7. How many penknives at $\$\frac{4}{7}$ each, can be bought for \$12? For \$6? For \$9? For \$11?

WRITTEN EXERCISES.

1. Divide 16 by $\frac{3}{4}$.

OPERATION.

$$16 \div \tfrac{3}{4} = \tfrac{64}{4} \div \tfrac{3}{4} = 21\tfrac{1}{3}.$$

Or,

$$16 \div \tfrac{3}{4} = \frac{16 \times 4}{3} = 21\tfrac{1}{3}.$$

SOLUTION. — Before dividing we reduce 16 to fourths. 16 is equal to $\frac{64}{4}$; which, divided by $\frac{3}{4}$, gives $\frac{64}{3} = 21\frac{1}{3}$.

Or, multiply 16 by the denominator 4, and divide the product by the numerator 3. This operation gives $\frac{64}{3} = 21\frac{1}{3}$, the required result.

Find the value of:

2. $21 \div \frac{3}{4}$.
3. $15 \div \frac{2}{7}$.
4. $11 \div \frac{5}{6}$.
5. $37 \div \frac{2}{3}$.
6. $29 \div \frac{2}{5}$.
7. $65 \div \frac{5}{8}$.
8. $78 \div \frac{3}{7}$.
9. $56 \div \frac{4}{5}$.
10. $18 \div \frac{8}{15}$.
11. $54 \div \frac{10}{18}$.
12. $49 \div \frac{14}{15}$.
13. $36 \div \frac{8}{17}$.
14. $90 \div \frac{15}{16}$.
15. $72 \div \frac{16}{21}$.
16. $63 \div \frac{21}{25}$.
17. $18 \div \frac{54}{35}$.
18. $605 \div 7\frac{1}{2}$.
19. $322 \div 1\frac{3}{5}$.
20. $124 \div 3\frac{3}{7}$.
21. $428 \div 3\frac{2}{3}$.
22. $350 \div 3\frac{3}{4}$.
23. $272 \div 6\frac{1}{2}$.
24. $837 \div 5$[illegible].
25. $982 \div 7$[illegible].

CASE III.

108. To divide a fraction by a fraction.

ORAL EXERCISES.

1. Divide $\frac{2}{7}$ by $\frac{3}{5}$.

SOLUTION.—Invert the terms of the divisor, and proceed as in multiplication. Thus, $\frac{2}{7} \div \frac{3}{5} = \frac{2}{7} \times \frac{5}{3} = \frac{10}{21}$, *Ans.*

I.

2. $\frac{1}{2} \div \frac{3}{4}$.
3. $\frac{5}{6} \div \frac{2}{3}$.
4. $\frac{3}{4} \div \frac{1}{8}$.
5. $\frac{4}{9} \div \frac{2}{3}$.
6. $\frac{7}{8} \div \frac{1}{4}$.
7. $\frac{1}{4} \div \frac{3}{5}$.
8. $\frac{2}{3} \div \frac{5}{6}$.
9. $\frac{2}{5} \div \frac{7}{9}$.
10. $\frac{8}{9} \div \frac{5}{6}$.
11. $\frac{3}{8} \div \frac{6}{9}$.
12. $\frac{2}{5} \div \frac{1}{4}$.
13. $\frac{8}{9} \div \frac{4}{6}$.
14. $\frac{5}{7} \div \frac{5}{8}$.
15. $\frac{7}{8} \div \frac{7}{8}$.
16. $\frac{1}{2} \div$ [illegible].
17. $\frac{5}{6} \div$ [illegible].

18. $\frac{5}{12} \div \frac{1}{2}$.
19. $\frac{9}{10} \div \frac{3}{5}$.
20. $\frac{14}{15} \div \frac{7}{8}$.
21. $\frac{8}{25} \div \frac{4}{5}$.
22. $\frac{2}{3} \div \frac{8}{15}$.
23. $\frac{6}{7} \div \frac{2}{14}$.
24. $\frac{1}{4} \div \frac{18}{25}$.
25. $\frac{5}{6} \div \frac{7}{24}$.
26. $3\frac{1}{2} \div 2\frac{1}{3}$.
27. $5\frac{1}{4} \div 1\frac{3}{4}$.
28. $2\frac{1}{6} \div 3\frac{1}{3}$.
29. $5\frac{1}{3} \div 1\frac{7}{9}$.
30. $8\frac{1}{2} \div 2\frac{1}{4}$.
31. $4\frac{2}{5} \div 1\frac{5}{6}$.
32. $7\frac{1}{2} \div 8\frac{3}{4}$.
33. $6\frac{3}{7} \div 1\frac{4}{5}$.

II.

1. How many pounds of butter can be bought for \$$\frac{3}{4}$, at \$$\frac{1}{4}$ a pound?

2. How many 6ths in a third? In a $\frac{1}{2}$?

3. How many pounds of cheese at \$$\frac{1}{8}$ a pound, can be bought for \$$\frac{1}{4}$? \$$\frac{1}{2}$? \$$\frac{5}{6}$? \$$\frac{3}{4}$? \$$\frac{2}{3}$?

4. How many $\frac{1}{4}$ pints can be put in a bottle containing $\frac{3}{2}$ pints?

5. How many pieces of ribbon $\frac{3}{7}$ of a yard long, can be cut from a piece $\frac{11}{12}$ of a yard long?

6. Sarah having $\frac{3}{4}$ of a pie, gave each of her sisters $\frac{3}{16}$ of a pie; how many sisters had she?

7. Divide the sum of $\frac{3}{8}$ and $\frac{1}{8}$ by their difference.

WRITTEN EXERCISES.

1. Divide $\frac{4}{7}$ by $\frac{3}{8}$.

OPERATION.

$$\frac{4}{7} \div \frac{3}{8} = \frac{32}{56} \div \frac{21}{56} = \frac{32}{21} = 1\frac{11}{21}.$$

Or,

$$\frac{3}{8} \text{ of quotient} = \frac{4}{7}$$

$$\frac{1}{8}, \quad `` \quad = \frac{4}{7 \times 3}$$

$$\frac{8}{8}, \text{ or } `` \quad = \frac{4 \times 8}{7 \times 3} = 1\frac{11}{21}.$$

SOLUTION. — Having reduced the two given fractions to a common denominator, divide their numerators, which will give $\frac{32}{21} = 1\frac{11}{21}$.

2. In this problem we have to find a quotient, which, when multiplied by the divisor $\frac{3}{8}$, will give the dividend $\frac{4}{7}$, for product. Hence, $\frac{3}{8}$ of the quotient is equal to $\frac{4}{7}$; $\frac{1}{8}$ of the quotient is equal to $\frac{1}{3}$ of $\frac{4}{7}$, or $4 \div (7 \times 3)$; and $\frac{8}{8}$, the quotient, is equal to 8 times $\frac{4}{7} \div (7 \times 3)$, or $(4 \times 8) + (7 \times 3)$, which is equal to $1\frac{11}{21}$.

2. If $\frac{3}{5}$ of a gallon of molasses cost $\frac{7}{8}$ of a dollar, how much will a gallon cost?

OPERATION.

$$\begin{aligned}
\tfrac{3}{5}\text{ gal.} &= \$\tfrac{7}{8}.\\
\tfrac{1}{5}\quad\text{“}\quad &= \frac{7}{8\times 3} = \$\tfrac{7}{24}.\\
\tfrac{5}{5}\quad\text{“}\quad &= \frac{7\times 5}{8\times 3} = \$1\tfrac{11}{24}.
\end{aligned}$$

SOLUTION.—If $\frac{3}{5}$ of a gallon cost $\$\frac{7}{8}$, $\frac{1}{5}$ of a gallon will cost $\frac{1}{3}$ of $\$\frac{7}{8}$, which is $\$\frac{7}{24}$; and $\frac{5}{5}$, or 1 gallon, will cost 5 times $\$\frac{7}{24}$, or $\$\frac{35}{24}$, which is equal to $\$1\frac{11}{24}$.

NOTE. — The solution of this problem affords a practical illustration of the following concise and valuable rule · “*Invert the terms of the devisor and proceed as in multiplication.*” But care must be taken that the pupils understand how to distinguish between the divisor and the dividend. To attain this end they should, at least in the beginning, be required to give a thorough analysis of every problem.

Find the value of:

3. $\frac{9}{11}\div\frac{3}{7}$
4. $\frac{16}{27}\div\frac{8}{9}$
5. $\frac{13}{31}\div\frac{7}{9}$
6. $\frac{18}{19}\div\frac{2}{3}$
7. $\frac{21}{25}\div\frac{4}{5}$
8. $\frac{23}{18}\div\frac{5}{6}$
9. $\frac{9}{20}\div\frac{1}{4}$
10. $\frac{1}{15}\div\frac{3}{4}$

11. $\frac{15}{16}\div\frac{9}{10}$
12. $\frac{24}{37}\div\frac{8}{15}$
13. $\frac{9}{11}\div\frac{15}{8}$
14. $\frac{21}{55}\div\frac{2}{11}$
15. $\frac{11}{31}\div\frac{12}{13}$
16. $\frac{28}{65}\div\frac{7}{16}$
17. $\frac{26}{27}\div\frac{13}{18}$
18. $\frac{34}{25}\div\frac{7}{30}$

19. $15\frac{3}{4}\div2\frac{1}{2}$.
20. $37\frac{1}{2}\div3\frac{1}{8}$.
21. $33\frac{1}{3}\div5\frac{1}{2}$.
22. $84\frac{1}{4}\div6\frac{1}{3}$.
23. $52\frac{1}{5}\div1\frac{5}{7}$.
24. $20\frac{5}{6}\div4\frac{8}{9}$.
25. $40\frac{2}{7}\div3\frac{3}{8}$.
26. $66\frac{2}{3}\div2\frac{7}{9}$.

27. $125\frac{1}{2}\div10\frac{1}{3}$
28. $214\frac{2}{3}\div20\frac{1}{2}$
29. $321\frac{1}{4}\div15\frac{2}{3}$
30. $140\frac{5}{8}\div19\frac{3}{8}$
31. $260\frac{6}{7}\div24\frac{1}{2}$
32. $300\frac{5}{8}\div16\frac{1}{4}$
33. $459\frac{2}{3}\div30\frac{3}{7}$
34. $283\frac{8}{9}\div60\frac{5}{6}$

35. $4163\frac{2}{5}\div33\frac{1}{3}$.
36. $6075\frac{1}{8}\div62\frac{1}{2}$.
37. $3982\frac{1}{2}\div25\frac{1}{4}$.
38. $5100\frac{1}{4}\div72\frac{3}{4}$.
39. $2095\frac{7}{8}\div30\frac{1}{8}$.
40. $1776\frac{3}{5}\div70\frac{1}{2}$.
41. $2612\frac{3}{7}\div72\frac{1}{7}$.
42. $1087\frac{1}{3}\div25\frac{1}{6}$.

COMPOUND AND COMPLEX FRACTIONS.

109. ***Reduction of Compound and Complex Fractions*** is the process of changing them to simple fractions.

CASE I.

110. To reduce compound fractions to simple ones.

1. What is the value of $\frac{5}{7}$ of $\frac{3}{4}$?

OPERATION.

$\frac{5}{7}$ of $\frac{3}{4} = \frac{5}{7} \times \frac{3}{4} = \frac{15}{28}$

SOLUTION.—$\frac{1}{7}$ of $\frac{3}{4}$ equals $\frac{3}{28}$; and since $\frac{1}{7}$ of $\frac{3}{4}$ equals $\frac{3}{28}$, $\frac{5}{7}$ of $\frac{3}{4}$ equals 5 times $\frac{3}{28}$, which gives $\frac{15}{28}$ for the required value of the given fractions.

WRITTEN EXERCISES.

What is the value of:

2. $\frac{2}{3}$ of $\frac{1}{5}$? *Ans.* $\frac{2}{15}$.
3. $\frac{4}{7}$ of $\frac{5}{6}$? *Ans.* $\frac{10}{21}$.
4. $\frac{6}{7}$ of $\frac{9}{10}$? *Ans.* $\frac{27}{35}$.
5. $\frac{11}{12}$ of $\frac{15}{16}$? *Ans.* $\frac{55}{64}$.
6. $\frac{18}{19}$ of $\frac{57}{72}$? *Ans.* $\frac{3}{4}$.
7. $\frac{7}{15}$ of $\frac{45}{91}$? *Ans.* $\frac{3}{13}$.

8. $\frac{4}{5}$ of $\frac{3}{7}$ of $\frac{7}{8}$?
9. $\frac{9}{10}$ of $\frac{5}{6}$ of $\frac{24}{35}$?
10. $\frac{7}{9}$ of $\frac{67}{77}$ of $\frac{112}{134}$?
11. $\frac{6}{27}$ of $\frac{64}{81}$ of $\frac{9}{27}$?
12. $\frac{117}{931}$ of $\frac{33}{89}$ of $\frac{7}{351}$?
13. $\frac{4}{13}$ of $\frac{26}{24}$ of $\frac{96}{3}$ of $\frac{39}{16}$?

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CASE II.

111. To reduce complex fractions to simple ones.

1. Reduce $\frac{\frac{7}{9}}{\frac{3}{5}}$ to a simple fraction.

OPERATION.

$$\frac{\frac{7}{9}}{\frac{3}{5}} = \frac{7}{9} \div \frac{3}{5} = \frac{7}{9} \times \frac{5}{3}$$
$$\frac{7}{9} \times \frac{5}{3} = \frac{35}{27} = 1\frac{8}{27}$$

SOLUTION.—This expression signifies that $\frac{7}{9}$ is to be divided by $\frac{3}{5}$. Hence, by applying the rule for division we have $\frac{7}{9} \times \frac{5}{3} = 1\frac{8}{27}$.

WRITTEN EXERCISES.

Reduce the following to simple fractions:

2. $\frac{\frac{3}{4}}{\frac{4}{7}}$. *3.* $\frac{\frac{11}{12}}{\frac{3}{4}}$. *4.* $\frac{\frac{15}{16}}{\frac{9}{10}}$. *5.* $\frac{\frac{25}{37}}{\frac{5}{6}}$.

6. $\frac{3\frac{1}{2}}{6\frac{1}{3}}$. *7.* $\frac{4\frac{1}{5}}{5\frac{1}{6}}$. *8.* $\frac{12\frac{3}{4}}{16\frac{5}{9}}$. *9.* $\frac{22\frac{2}{3}}{30\frac{3}{4}}$.

10. $\frac{\frac{4}{7} \times \frac{14}{15}}{\frac{32}{45} \times \frac{7}{8}}$. *11.* $\frac{\frac{10}{17} \times \frac{17}{35}}{\frac{19}{28} \times \frac{3}{19}}$. *12.* $\frac{\frac{89}{105} \times \frac{35}{89}}{\frac{6}{13} \times \frac{39}{48}}$.

13. $\frac{\frac{2}{3} \times \frac{5}{6}}{3\frac{1}{4} \times 1\frac{1}{2}}$. *14.* $\frac{14\frac{1}{2}}{2} \times \frac{6\frac{1}{3}}{5\frac{1}{7}}$. *15.* $\frac{\frac{8\frac{1}{4}}{5} \times \frac{9}{10}}{\frac{20}{9\frac{1}{7}} \times \frac{12\frac{1}{3}}{2\frac{1}{3}}}$.

MENTAL EXERCISES.

1. If 2 pounds of tea cost \$$\frac{5}{8}$, what will 1 pound cost?

2. How much butter can be bought for \$8 at \$$\frac{2}{5}$ a pound?

3. Joseph divided $\frac{8}{9}$ of a melon equally between his two sisters; what part did each receive?

4. At \$$\frac{4}{7}$ a yard, how many yards of silk will \$32 buy?

5. If a train run $\frac{9}{14}$ of a mile a minute, how long will it take for a trip of 50 miles?

6. How many bushels of oats worth \$$1\frac{1}{3}$ a bushel, will pay for some hay worth \$63?

7. If you divide $16\frac{2}{3}$ barrels of flour among 7 poor families, how much does each family receive?

8. When potatoes are worth \$$\frac{5}{8}$ per bushel, how many bushels can be bought for \$$27\frac{1}{2}$?

9. If $23\frac{1}{3}$ barrels of sugar cost \$120, what will one barrel cost?

10. If a turkey cost $9\frac{1}{2}$ shillings, how many turkeys can be bought for $47\frac{1}{2}$ shillings?

11. How long will it take a horse to eat $\frac{11}{12}$ of a bushel of oats, if he eat $\frac{4}{7}$ of a bushel in a day?

12. What will 1 dozen of eggs cost, if $26\frac{1}{4}$ dozen cost $\$10\frac{1}{2}$?

13. A gentleman having $\frac{9}{10}$ of a factory, divided his share equally among his 5 children; what did each child receive?

14. If $\frac{8}{9}$ of a yard of broadcloth cost $\$2\frac{1}{2}$, what will 1 yard cost?

15. How many men will be required to do in 1 day, a piece of work which 9 men can do in $6\frac{1}{3}$ days?

WRITTEN EXERCISES.

1. If $32\frac{2}{3}$ ounces of silver cost $\$27\frac{1}{2}$, what is the cost of 1 ounce? *Ans.* $\$0.84\frac{9}{49}$.

2. If $\$48\frac{1}{6}$ will buy $56\frac{2}{3}$ bushels of wheat, how many bushels can be bought for $\$95\frac{5}{8}$? *Ans.* $112\frac{1}{2}$ bushels.

3. What will 1 barrel of sugar cost, if $52\frac{3}{4}$ barrels cost $\$461\frac{9}{16}$? *Ans.* $\$8\frac{3}{4}$.

4. Bought $342\frac{1}{4}$ acres of land for $\$8605\frac{1}{7}$; what was that an acre? *Ans.* $\$25\frac{1}{7}$.

5. If $39\frac{1}{4}$ tons of Lehigh coal cost $\$215\frac{7}{8}$, what is the price per ton? *Ans.* $\$5\frac{1}{2}$.

6. If $\frac{16}{25}$ of a house be valued at \$21800, what is the value of the whole house? *Ans.* $\$34062\frac{1}{2}$.

7. At the rate of $\$35\frac{1}{2}$ for $9\frac{7}{8}$ yards of cloth, how many yards can be bought for $\$88\frac{3}{4}$? *Ans.* $24\frac{11}{16}$.

8. The greater of two numbers is 2739, and their product is 27; what is the less? *Ans.* $\frac{9}{913}$.

9. The product of two numbers is 442 and one of them is $52\frac{4}{7}$; what is the other? *Ans.* $8\frac{75}{184}$.

10. A grocer sold 36 bushels of potatoes at $62\frac{1}{2}$ cents a bushel, and received in payment, eggs worth 45 cents a dozen. How many eggs did he get? *Ans.* 50 dozen.

11. If 25 men consume $\frac{5}{7}$ of 1400 pounds of meat in a month, how much does 1 man consume in the same time? *Ans.* 40 pounds.

12. A boy divided 306 apples among his companions, giving $19\frac{1}{8}$ apples to each. How many companions had he? *Ans.* 16.

13. Henry Perkins bought land for $85340, and sold it so as to gain $\frac{5}{17}$ of the cost, or $25 an acre; how many acres did he buy? *Ans.* 1004 acres.

14. A person having $6500, distributed $\frac{8}{13}$ of it among some poor families, giving each family $\$181\frac{9}{11}$. How many poor families were there? *Ans.* 22.

MENTAL REVIEW.

I.

1. 56 is $\frac{7}{8}$ of what number?

OPERATION.

$\frac{7}{8} = 56$
$\frac{1}{8} = 8$
$\frac{8}{8} = 64$

SOLUTION.—Since $\frac{7}{8}$ of a certain number is 56, $\frac{1}{8}$ of the number must be $\frac{1}{7}$ of 56, which is 8. And if $\frac{1}{8}$ of the number be 8, $\frac{8}{8}$ of the number, or the number itself, must be 8 times as much as $\frac{1}{8}$ or 8 times 8, which are 64. Therefore, 56 is $\frac{7}{8}$ of 64.

2. 126 is $\frac{14}{15}$ of what number?
3. $16\frac{1}{3}$ is $\frac{1}{4}$ of what number?
4. 36 is $\frac{4}{7}$ of what number?
5. 60 is $\frac{9}{10}$ of what number?
6. $15\frac{1}{5}$ is $\frac{19}{20}$ of what number?
7. 96 is $\frac{12}{31}$ of what number?
8. $17\frac{1}{2}$ is $\frac{5}{9}$?
9. $19\frac{1}{3}$ is $\frac{2}{3}$?
10. 13 is $\frac{9}{8}$?
11. 25 is $\frac{3}{5}$?
12. 48 is $\frac{3}{4}$?
13. 56 is $\frac{7}{13}$?

II.

1. 63 is $\frac{9}{10}$ of how many times 5?

OPERATION.	
$\frac{9}{10} = 63$	**SOLUTION.**—If $\frac{9}{10}$ of a certain number of times 5 be 63, $\frac{1}{10}$ of that number of times 5, must be $\frac{1}{9}$ of 63, or 7; and $\frac{10}{10}$ of that number of times 5 is 10 times 7, or 70. Hence, 70 is as many times 5 as 5 is contained times in 70, which is 14 times. Therefore, 63 is $\frac{9}{10}$ of 14 times 5.
$\frac{1}{10} = 7$	
$\frac{10}{10} = 70$	
$70 \div 5 = 14.$	

2. 40 is $\frac{5}{6}$ of how many times 16? 32? 64? 48?

3. 75 is $\frac{3}{11}$ of how many times 25? 75? 50? 100?

4. 27 is $\frac{3}{16}$ of how many times 72? 24? 48? 16?

5. 42 is $\frac{14}{17}$ of how many times 3? 17? 51? 34?

6. $12\frac{1}{2}$ is $\frac{5}{4}$ of how many times 60? 20? 30? 40?

7. 60 is $\frac{1}{2}$ of how many times $\frac{2}{3}$ of 90? 60? 120?

8. $3\frac{3}{4}$ is $\frac{1}{8}$ of how many times $\frac{4}{5}$ of 25? 40? 50?

9. $1\frac{1}{2}$ is $\frac{1}{10}$ of how many times $\frac{1}{2}$ of 100? 50? 40?

10. 84 is $\frac{12}{13}$ of how many times $\frac{2}{3}$ of $10\frac{1}{2}$? 42? 30?

11. 18 is $\frac{1}{4}$ of how many times $\frac{2}{5}$ of 15? 25? 45?

12. $\frac{6}{7}$ is $\frac{3}{8}$ of how many times $\frac{1}{4}$ of $9\frac{1}{7}$? $18\frac{2}{7}$? $4\frac{4}{7}$?

III.

1. $\frac{4}{7}$ of 84 is $\frac{8}{9}$ of what number? $\frac{6}{7}$ of what number?

2. $\frac{4}{5}$ of 30 is how many twelfths of 72? 48? 96?

3. $\frac{8}{9}$ of 27 is $\frac{6}{7}$ of how many times 6? 7? 8? 9?

4. $\frac{3}{4}$ of 32 are $\frac{6}{7}$ of 21 times what number? $\frac{2}{3}$ of 30?

5. $\frac{4}{9}$ of $\frac{6}{8}$ of 42 is 7 times what part of 120?

6. At the rate of 8 oranges for 17 cents, how many oranges could you buy for 85 cents?

7. If 9 bushels cost $2, what will 14 bushels cost?

8. In what time can 12 men do a piece of work for which 4 men require 13 days?

9. Sold a horse for \$80, which was only $\frac{8}{9}$ of its cost; what did I lose?

10. If $\frac{6}{7}$ of a barrel of flour is worth \$12, what will $4\frac{1}{2}$ barrels be worth?

11. If a boy earn \$$17\frac{2}{3}$ in 4 weeks, what will he earn in $13\frac{1}{2}$ weeks?

12. How much less than \$15 will 10 yards of satin cost, if 5 yards cost \$$7\frac{1}{3}$?

13. If a horse travel $27\frac{3}{4}$ miles in 7 hours, how far will he travel in 5 hours?

14. A man having $24\frac{3}{4}$ bushels of potatoes, sold $\frac{2}{3}$ of them; how many bushels had he left?

15. If $\frac{1}{3}$ of a box of raisins cost \$$1\frac{1}{5}$, what will $\frac{6}{7}$ of a box cost?

16. How many pounds of sugar worth $13\frac{5}{7}$ cents can be had for $4\frac{4}{5}$ pounds of butter at 20 cents a pound?

17. Henry and William together picked 8 times $4\frac{7}{8}$ quarts of blackberries, and shared them equally with their companions. How many companions had they, if each received $3\frac{1}{4}$ quarts?

18. Bought 32 yards of cloth for \$96; but, being damaged, I sold $\frac{5}{8}$ of it at \$$2\frac{1}{3}$ a yard, and the remainder for what it cost. How much did I lose?

19. How many chestnuts will pay for 25 walnuts, if 9 walnuts are worth $16\frac{1}{5}$ chestnuts?

20. A man bought 35 sheep, at the rate of 10 for \$$14\frac{2}{7}$. How much did they cost?

21. A boy gave $13\frac{1}{2}$ cents for oranges at the rate of 15 oranges for $22\frac{1}{2}$ cents. How many did he buy? How much would 20 oranges cost at the same rate?

22. Grafton is 162 miles from Albany, and $\frac{5}{9}$ of this distance is 2 miles less than $\frac{1}{3}$ of 3 times the distance to Westfield. How far is it to Westfield?

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23. The insurance of a ship amounted to \$1200, and $\frac{1}{3}$ of that is $\frac{1}{20}$ of 8 times the value of the cargo; what is the value of the cargo?

24. If a sleigh cost \$150, what would be the cost of a wagon, if $\frac{3}{5}$ of the cost of the sleigh were $\frac{5}{11}$ of twice the cost of the wagon?

25. A coat cost \$30; the $\frac{4}{5}$ of which is $\frac{2}{3}$ of 12 times the price of a hat. What is the price of a hat?

26. A boy being asked his age, said that 27 years equalled $\frac{3}{4}$ of 3 times his age. How old was he?

27. Mr. Brighton gave \$45$\frac{1}{3}$ for a cow, which was $\frac{1}{2}$ as much as Mr. Anthony paid for his. How much did both cows cost?

28. A man after spending $\frac{2}{3}$ of his fortune, found that \$500 was $\frac{5}{16}$ of what he had remaining. What was his fortune?

29. If a 4-pound loaf cost 13 cents when flour is \$6$\frac{1}{2}$ a barrel, what will it cost when flour is \$9 a barrel?

30. To $\frac{3}{4}$ of a gross, add $\frac{4}{5}$ of a hundred, and subtract the sum from $\frac{7}{25}$ of a thousand.

31. If it require \$45 worth of provisions to serve 12 men 3 days, how many dollars' worth will serve 3 men 6 days?

32. Philip spent $\frac{5}{8}$ of all his money, and gave the remainder for 15 yards of cloth at \$2$\frac{2}{5}$ a yard. How much had he at first?

33. When A and B engaged in play, A's money was $\frac{4}{5}$ of B's. B gained \$25, which was $\frac{1}{5}$ of 2$\frac{1}{3}$ times as much as he commenced with. How much had A left?

34. A merchant, after selling from a cask of wine 12 gallons more than $\frac{1}{4}$ of the whole, found that the number of gallons left was just twice the number of gallons sold. How many gallons did the cask contain at first?

35. I sold 2 barrels of flour for $\$15\frac{3}{5}$, which is $\frac{1}{5}$ of what I received for all I had left, at $\$6\frac{1}{2}$ a barrel; how many barrels did I sell in all?

36. Thomas has 75 cents and James 63. Thomas spent $\frac{2}{5}$ of his, and James lost $\frac{4}{7}$ of his; what part of Thomas's money then equalled James's?

WRITTEN REVIEW.

1. A gentleman divides $5000 into 5 equal shares, and gives $3\frac{1}{2}$ of these shares to a benevolent society. What amount has he left? *Ans.* $1500.

2. How many pounds of coffee at $19\frac{1}{2}$ cents a pound, must be given for $23\frac{2}{5}$ pounds of butter at $24\frac{1}{2}$ cents a pound? *Ans.* $29\frac{2}{5}$ pounds.

3. A farmer gave $\frac{3}{7}$ of all his money for a horse, $\frac{1}{8}$ of it for a cow, and had $175 left. How much had he at first? *Ans.* $392.

4. I have $563\frac{8}{9}$ acres of land. How much will be left after selling $\frac{6}{7}$ of $\frac{4}{5}$ of it? *Ans.* $177\frac{2}{9}$ acres.

5. How many pairs of sheets requiring $4\frac{2}{9}$ yards can be made from a piece of sheeting containing 54 yards, leaving a remainder of $11\frac{7}{9}$ yards? *Ans.* 10 pairs.

6. If a boy earn $\$32\frac{1}{4}$ in $\frac{1}{5}$ of a year, how much will he earn in 8 months? *Ans.* $107.50.

7. The difference of two numbers is $47\frac{3}{21}$, and their sum is $183\frac{5}{7}$; what are the numbers? *Ans.* $68\frac{2}{7}$ and $115\frac{3}{7}$.

8. Mr. Bultitude's stock increased by $\frac{7}{8}$ of its value plus $\$484\frac{3}{10} = \$6785$; what is the value of his stock? *Ans.* $\$3360\frac{28}{75}$.

9. A mechanic received \$$1.25\frac{1}{2}$ a day for a certain time. After paying his board and other expenses with the $\frac{3}{5}$ of his earnings, he had \$15.06 left. How many days did he work? *Ans.* 30 days.

10. Two pipes will fill a cistern in 62 minutes, and one of them in 93 minutes. In what time will the other fill it? *Ans.* 186 minutes.

11. A and B can do a certain piece of work in 7 days; A and C, in 9 days; and B and C, in 6 days. In what time will all working together do it? *Ans.* $4\frac{40}{53}$ days.

12. Matthew agreed to work 32 days for \$21; but was obliged to leave off after $13\frac{1}{4}$ days. What part of the \$21 did he earn? *Ans.* $\frac{53}{128}$ of it.

13. Sold 1640 bushels of wheat which cost $62\frac{1}{2}$ cents a bushel, for 49 cents a bushel. What was the loss? *Ans.* \$221.40.

14. If a carpenter earn \$3.50 a day, how many days will he have to work to pay for a suit of clothes, of which the coat costs \$$28\frac{1}{3}$, the pants \$9.75, and the vest \$$4\frac{1}{2}$? *Ans.* $12\frac{1}{6}$ days.

15. Exchanged 63 tubs of butter containing each $37\frac{4}{9}$ pounds, at $22\frac{1}{2}$ cents a pound, for 50 barrels of flour at \$$8\frac{1}{4}$ per barrel, and received the balance in cash. What was the balance? *Ans.* \$$118.27\frac{1}{2}$.

16. Sold my house and farm of $85\frac{3}{7}$ acres for \$8132. Allowing \$2750 for the house, what did I receive per acre for the land? *Ans.* \$63.

17. If a man take $\frac{2}{5}$ of a day to do a certain piece of work, how long will it take him to do $\frac{3}{4}$ of $\frac{5}{8}$ of it? *Ans.* $\frac{3}{16}$ of a day.

18. If 2 be added to both terms of the fraction $\frac{4}{5}$, will the value of the fraction be increased or diminished?

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19. A merchant sold 43 barrels of flour for \$$506\frac{2}{3}$, which was $\frac{5}{6}$ as much as he received for all he had left at \$8 a barrel. How many barrels did he sell in all?

Ans. 119 barrels.

20. A and B bought 15900 acres of land, of which B was to have 7 acres as often as A 8. How many acres did each buy? *Ans.* A, 8480; B, 7420.

21. If 2 be subtracted from both terms of the fraction $\frac{4}{5}$, will the value of the fraction be increased or diminished?

Ans. Diminished $\frac{2}{15}$.

22. Will the value of the fraction $\frac{5}{4}$, be increased or diminished if 2 be added to both of its terms? If 2 be subtracted from both terms?

23. Two carpenters can do a piece of work in 37 days; the first can do $\frac{7}{8}$ as much as the second. In what time can each do it? *Ans.* $69\frac{3}{8}$ and $79\frac{2}{7}$ days.

24. The sum of two numbers is 75840, and their difference is equal to $\frac{2}{3}$ of the less. What are the numbers?

25. How many yards of carpet, at \$$1.87\frac{1}{2}$ a yard, can I get for 200 quarts of strawberries at 27 cents a quart?

Ans. $28\frac{4}{5}$ yards.

26. The difference between $\frac{3}{4}$ and $\frac{2}{3}$ of a number is 20 more than $\frac{5}{84}$ of the number. What is the number?

Ans. 840.

27. A capitalist invested $\frac{3}{8}$ of his capital in flour, $\frac{2}{5}$ of the remainder in real estate, and the remainder, \$22560, in stocks. What was the amount of his capital?

Ans. \$60160.

28. The sum of three numbers is $60\frac{1}{12}$; the greatest is $30\frac{1}{4}$, and the least is $12\frac{1}{3}$. Find their product.

29. A woman bought eggs at the rate of 3 for 5 cents, and sold them at $2\frac{1}{2}$ cents apiece, thereby gaining \$1.00. *How many did she buy?* *Ans.* 120.

DECIMAL FRACTIONS.

112. A ***Decimal Fraction*** is a fraction whose denominator is 10, or some higher power of 10. Thus, $\frac{7}{10}$, $\frac{19}{100}$, $\frac{23}{1000}$, etc., are decimal fractions.

113. ***Decimals,*** or *decimal fractions,* are like common fractions, except that their denominators increase and decrease by the uniform ratio of 10. Thus, the common fraction $\frac{18}{100}$, is the same as the decimal .18.

114. A ***Mixed Decimal*** is a mixed number whose fractional part is a decimal; as, 9.31.

NOTATION.

115. To express a decimal fraction, we place a point before the numerator and omit the denominator.

116. When integers and decimals are written together, the former are placed on the left and the latter on the right of the decimal point, as shown in the following

NUMERATION TABLE.

Ten-millions.	Millions.	Hund.-thousands.	Ten-thousands.	Thousands.	Hundreds.	Tens.	Units.	Decimal Point.	Tenths.	Hundredths.	Thousandths.	Ten-thousandths.	Hund.-thousandths.	Millionths.	Ten-millionths.	Etc. Etc.
9	6	8	1	5	7	2	4	.	3	4	5	2	7	9	6	

Integers. Decimals.

WRITTEN EXERCISES.

1. Write six-tenths, as a decimal.

2. Write forty-seven hundredths.

3. Write twenty-five thousandths.

SOLUTION.—In number 1, the numerator, 6, is written as in simple numbers, and the denominator, tenths, is indicated by placing the decimal immediately before it; thus, .6.

In number 2, since 47 hundredths is equal to 4 tenths and 7 hundredths, we write 4 in tenths' place and 7 in hundredths' place, putting the point before the 4; thus, .47.

In number 3, because 25 thousandths is the same as 0 tenths, 2 hundredths, 5 thousandths, we write 0 in tenths' place, 2 in hundredths', and 5 in thousandths', placing the point before the naught; thus, .025.

4. Eighty-nine thousandths. *Ans.* .089.

5. Three hundred four ten-thousandths.

6. One hundred sixty-three millionths.

7. Nine hundred nineteen thousandths.

8. Seventy-six ten-millionths. *Ans.* .0000076.

9. Five hundred two hundred-thousandths.

10. Forty-two hundred-millionths.

11. Fourteen thousandths. *Ans.* .014.

12. Eight hundredths. *Ans.* .08.

NOTE.—When an integer and a decimal occur in the same written number, a comma should always be placed after the integer, and the decimal part should be introduced by the word and. Thus, 200.004 should be written two hundred, and four thousandths; but .204 two hundred four thousandths.

13. Fifty-eight, and ten hundredths.

14. One thousand three, and sixteen thousandths.

15. Seventeen thousand nine hundred, and ten hundred-thousandths.

16. Thirty, and five hundred seven millionths.

17. Three hundred three million, and two thousand seventy ten-millionths.

18. Sixty-one thousand, and eleven hundredths.

19. Seventy-five thousand, and ninety-six hundred-thousandths.

20. Twelve million, and twelve hundred-millionths.

21. Six million sixty thousand, and two hundred sixty-four millionths.

22. Thirty billion one hundred thousand thirty, and three thousand two hundred six hundred-millionths.

23. 8 million 40 thousand 9 hundred 1, and 901 hundred-thousandths.

NUMERATION.

WRITTEN EXERCISES.

Read and write the following: *1.* .83. *2.* .057.

Solution.—In number 1, .83 expresses 8 tenths and 3 hundredths; or, since 8 tenths is equal to 80 hundredths, and 80 hundredths plus 3 hundredths equal 83 hundredths, this decimal may also be read 83 hundredths.

In number 2, .057 expresses 5 hundredths and 7 thousandths; or, as 5 hundredths is equal to 50 thousandths, and 50 thousandths plus 7 thousandths equal 57 thousandths, it is also read fifty-seven thousandths.

3. .1298.
4. .0204.
5. .0039.
6. .00204.
7. .50608.
8. .01829.

9. .8644.
10. .420369.
11. .0047812.
12. 240.1800.
13. 542.0871.
14. 300.0094.

15. 1408.00016.
16. 871914.00809.
17. 500001.0000051.
18. 9000000.0000029.
19. 50047.080060037.
20. 130900.00041396.

II.

21. .7854.
22. .0034.
23. .0169.
24. .5236.
25. 3.1416.
26. .07958.
27. 4.1888.
28. 1.4142.
29. .433013.
30. 277.274.
31. .4971499.
32. 15.198684.

REDUCTION.

CASE I.

117. To reduce a common fraction to a decimal.

WRITTEN EXERCISES.

1. Reduce $\frac{7}{8}$ to a decimal.

OPERATION.

$\frac{7}{8} = 7 \div 8 =$
$8\,\underline{)\,7000}$
$875 = .875.$

SOLUTION.—Since the value of a fraction is equal to the quotient obtained by dividing the numerator by the denominator, the value of $\frac{7}{8}$ is equal to 7 ÷ 8. To find this quotient we annex three ciphers to 7, which is equivalent to multiplying it by 1000. But the result obtained by dividing 7000 by 8 is 1000 times the true value of the fraction. We, therefore, divide this result, or quotient, by 1000, which is done by pointing off three decimal places. Hence, the common fraction, $\frac{7}{8}$, is equal to the decimal, .875.

Reduce the following to equivalent decimals:

2. $\frac{4}{5}$.	*Ans.* .8.	*10.* $23\frac{19}{25}$.	*Ans.* 23.76.
3. $\frac{14}{15}$.	*Ans.* .933 +.	*11.* $\frac{11}{12}$.	*Ans.* .9166 +.
4. $\frac{15}{16}$.	*Ans.* .9375.	*12.* $15\frac{35}{75}$.	*Ans.* 15.466 +.
5. $\frac{3}{4}$.	*Ans.* .75.	*13.* $203\frac{5}{12}$.	*Ans.* 203.4166 +.
6. $\frac{18}{20}$.	*Ans.* .9.	*14.* $\frac{16}{17}$.	*Ans.* .9411 +.
7. $\frac{114}{120}$.	*Ans.* .95.	*15.* $\frac{129}{1500}$.	*Ans.* .086.
8. $\frac{5}{1250}$.	*Ans.* .004.	*16.* $501\frac{32}{33}$.	*Ans.* 501.9696 +.
9. $\frac{9}{625}$.	*Ans.* .0144.	*17.* $96\frac{13}{19}$.	*Ans.* 96.6842 +.

Case II.

118. To reduce a decimal to a common fraction.

WRITTEN EXERCISES.

1. Reduce .025 to a common fraction.

OPERATION.

$.025 = \frac{25}{1000} = \frac{1}{40}$.

SOLUTION.—In this decimal, the numerator is 25, and the denominator is 1000; therefore, the fractional expression is $\frac{25}{1000}$; which, reduced to its lowest terms, gives $\frac{1}{40}$. Hence, $.025 = \frac{1}{40}$.

2. Reduce the complex decimal, $.15\frac{1}{4}$, to an equivalent common fraction.

OPERATION.

$$.15\frac{1}{4} = \frac{15\frac{1}{4}}{100} = \frac{\frac{61}{4}}{100}$$

$$\frac{\frac{61}{4}}{100} = \frac{61}{400}, \textit{ Ans.}$$

SOLUTION.—The mixed decimal, $15\frac{1}{4}$, is $15\frac{1}{4}$ hundredths, which expressed as a common fraction, becomes $\frac{61}{400}$.

Change the following to equivalent common fractions:

3. $.62\frac{1}{2}$. *Ans.* $\frac{5}{8}$.
4. .125. *Ans.* $\frac{1}{8}$.
5. .05. *Ans.* $\frac{1}{20}$.
6. .1260. *Ans.* $\frac{63}{500}$.
7. .0255. *Ans.* $\frac{51}{2000}$.
8. $.008\frac{1}{3}$. *Ans.* $\frac{1}{120}$.
9. $.0033\frac{1}{3}$. *Ans.* $\frac{1}{300}$.
10. .1872. *Ans.* $\frac{117}{625}$.
11. $.0142\frac{1}{2}$. *Ans.* $\frac{57}{4000}$.
12. 24.075. *Ans.* $24\frac{3}{40}$.
13. $184.037\frac{1}{2}$ *Ans.* $184\frac{3}{80}$.
14. $67.0093\frac{1}{3}$. *Ans.* $67\frac{7}{750}$.
15. $.008\frac{5}{6}$. *Ans.* $\frac{53}{6000}$
16. .93750. *Ans.* $\frac{15}{16}$
17. .3125. *Ans.* $\frac{5}{16}$
18. $12.0018\frac{3}{4}$. *Ans.* $12\frac{3}{1600}$
19. 17.00125. *Ans.* $17\frac{1}{800}$
20. 2.09125. *Ans.* $2\frac{73}{800}$

21. $.0087\frac{1}{2}$.
22. $.1265\frac{1}{3}$.
23. $.2537\frac{1}{2}$.
24. $.7001\frac{1}{8}$.
25. .88048.
26. .20625.
27. .00058.
28. .15250.
29. 25.1850.
30. 70.9040.
31. 48.0080.
32. 40.6009.

ADDITION.

119. ***Addition of Decimals*** is the process of finding the sum of two or more decimals.

WRITTEN EXERCISES.

1. Find the sum of 15.09, 435.6082, .549, and 3.6.

OPERATION.

15.09
435.6082
.549
3.6
454.8472

SOLUTION.—Write the numbers so that units of the same order shall stand in the same column. This will bring all the decimal points in the same column. Then beginning at the right, add each column separately, setting down and carrying as in simple numbers. The decimal point in the sum must be in the same column with those of the given numbers.

Find the sum:

2. Of .524, .16, .0623, .800039, .0000403, .508, 3.002, $60\frac{5}{7}$, .12900, and 912.00972. *Ans.* 977.909385.

3. Of 708.53, 6.007, 904.0031, .000082, and 60.00005.

4. Of 80.016, 391.25, .0100096, and 8427.63050.

5. Add 90, and seven-tenths; six hundred, and 36 thousandths; forty-nine thousand, and 2 ten-thousandths; and 50 millions. *Ans.* 50049690.7362.

6. Find the sum of $\frac{4}{80}$, $\frac{5}{7}$, $\frac{6}{50}$, $\frac{25}{425}$, and $\frac{94}{125}$ in decimals, correct to the fourth place. *Ans.* 1.6951.

7. Add $18\frac{3}{4}$, $\frac{19}{25}$, $.65\frac{2}{5}$, 44.05, $83\frac{4}{25}$, $\frac{9}{35}$, and $210.06\frac{1}{2}$.

8. How many rods of fence will enclose a field the sides of which are, respectively, 97.185, $75\frac{1}{15}$, $134.002\frac{1}{2}$, and $144\frac{7}{8}$ rods? *Ans.* 451.7958 rods.

9. Add $3\frac{1}{3}$, $.6\frac{2}{3}$, $4.2\frac{1}{2}$, $.44\frac{4}{9}$, and $3.875\frac{5}{6}$.

10. Find the sum of $12\frac{3}{16}$, 480.625, $37\frac{2}{9}$, 62.5, $222.22\frac{2}{9}$, 6.1250, and $3864.5\frac{5}{9}$.

11. How many bushels of corn in four bins, containing, respectively, 16.24, 24.5, $130\frac{5}{8}$, and $64\frac{1}{10}$ bushels?

SUBTRACTION.

120. ***Subtraction of Decimals*** is the process of finding the difference between two decimals.

WRITTEN EXERCISES.

1. Subtract 6.0451 from 129.852.

OPERATION.

Minuend,	129.852
Subtrahend,	6.0451
Remainder,	123.8069

SOLUTION.—We write the subtrahend under the minuend so that units of the same order shall stand in the same column. If both decimals have not the same number of places, we annex ciphers or suppose them to be annexed, and then proceed as in subtraction of simple numbers. Place the decimal point in the remainder in the same column as that of the minuend and subtrahend.

2. From 9040.187 take 867.348. *Ans.* 8172.839.

3. From 884.13 take 197.487. *Ans.* 686.643.

4. From 8431.00194 take 769.100086.

5. From 4.009 take 3.641963. *Ans.* .367037.

6. From $.5\frac{1}{2}$ take five hundred-thousandths.

7. From 1 million take 9 millionths.

8. From $42\frac{5}{9}$ take $.93\frac{5}{16}$. *Ans.* 41.622430.

9. From 11.00011 take 1111 hundred-thousandths.

10. What is the difference between fourteen million one, and nine million sixteen ten-millionths?

11. From the sum of $5.16\frac{1}{4}$ and 38.5 take the difference between $33.3\frac{1}{3}$ and 20.05.

12. Find the difference between nine thousand twelve hundred-thousandths and 2.

13. From eighty-four, and twenty-five thousandths yards of cloth, 39, and eight hundredths yards were cut. How many yards remained?

MULT

121. *Multiplica* of finding the produc mals.

WRITT

1. Multiply 8.29 by

OPERATION.	SOLUTION.
8.29	alent comm
.43	$\frac{829}{100} \times \frac{43}{100}$ =
2487	gers; and sin
3316	produce ten
	four decimal
3.5647	those in the n

2. Multiply 62.18 by .2

3. Multiply 127.05 by 1.

4. Multiply 6.0009 by

Multiply:

5. 748.136 by 7.004.

6. 2.014 by .735.

7. 596.10 by 33.

8. 6.35 by 87⅝.

13. What is the value of

14. What is the value of

15. What is the product

16. Find the value of 3.4

17. What is the area of a

18. At 9.75 dollars per barrels of flour?

19. What must I pay for yard, and 29.125 yards at

DIVISION.

122. ***Division of Decimals*** is the process of nding the quotient when one or both terms are decimals.

WRITTEN EXERCISES.

1. Divide 9.5 by .625.

OPERATION.

625,) 9,5000 (15.2
6′25
3 250
3 125
1250
1250

SOLUTION.—As the dividend must contain, at least, as many decimal places as the divisor, we must, in this problem, annex two ciphers to the dividend. Then dividing as in simple numbers, we find a remainder of 125; to which we annex another cipher and continue the division. This gives a final quotient of 152. Now, since the dividend is the product of the quotient by the divisor, we subtract the number of decimal places in the divisor from the number in the dividend, and the remainder will be the number required for the quotient. Counting the three ciphers annexed, we have four decimal places in the dividend and three in the divisor. Hence, there will be one in the quotient.

Divide:

2. 70.4831 [illegible]5.802.
3. 3414.5 [illegible] 00.
4. 101.6688 by 43.08.
5. 71.1022 by 18.86.
6. 4171.15 by 1000.
7. 1.5108 by .019.
8. 17.1031 by .00053.
9. 125 by 2480.
10. .004 by .0016.
11. .000625 by 250.
12. .051 by .0015.
13. .001 by 100.
14. .03 by 18¾.
15. .56⅔ by 4.076.

16. $(81.96 \times \frac{3}{4}) \div (18\frac{1}{2} - 12.25) = ?$
17. $(47.008 + .125) \times (810 \div .018) = ?$
18. $300.163\frac{5}{8} \times (80.005 \div 16.001) - 900.0000099 = ?$

BUSINESS OPERATIONS.

123. In ***Business Operations*** there are generally three things considered; viz., the *quantity*, the *price*, and the *cost*.

124. The ***Quantity*** is the amount of anything bought or sold.

125. The ***Price*** is the value of a unit of the quantity.

126. The ***Cost*** is the value of the entire quantity.

127. An ***Aliquot Part*** of a number is one of the equal parts into which the number can be divided.

128. The principal aliquot parts of a dollar are as follows:

TABLE.

5 cents	=	$\frac{1}{20}$ of $1.00.	$6\frac{1}{4}$ cents	=	$\frac{1}{16}$ of $1.00.
10 cents	=	$\frac{1}{10}$ of $1.00.	$8\frac{1}{3}$ cents	=	$\frac{1}{12}$ of $1.00.
20 cents	=	$\frac{1}{5}$ of $1.00.	$12\frac{1}{2}$ cents	=	$\frac{1}{8}$ of $1.00.
25 cents	=	$\frac{1}{4}$ of $1.00.	$16\frac{2}{3}$ cents	=	$\frac{1}{6}$ of $1.00.
50 cents	=	$\frac{1}{2}$ of $1.00.	$33\frac{1}{3}$ cents	=	$\frac{1}{3}$ of $1.00.
75 cents	=	$\frac{3}{4}$ of $1.00.	$66\frac{2}{3}$ cents	=	$\frac{2}{3}$ of $1.00.

CASE I.

129. To find the cost of any quantity when the price of a unit is an aliquot part of a dollar.

1. What will 75 bushels of wheat cost at 50 cents a bushel?

OPERATION.
0 cts. = $½.
2) $75
$37½.

SOLUTION.—If it cost $1 per bushel, the cost would be as many dollars as there are bushels. But since the price is ½ of a dollar a bushel, the whole cost will be ½ as many dollars as there are bushels; or ½ of $75 = $75 ÷ 2 = $37½.

WRITTEN EXERCISES.

2. What cost 1297 dozen of eggs at 16⅔ cents a dozen?

3. At 6¼ cents a spool, what cost 9245 spools of thread?

4. What cost 7842 yards of muslin at 33⅓ cents a yard?

5. What is the cost of 525 pounds of sugar at 12½ cents pound? *Ans.* $65.625.

6. Find the cost of 2500 melons at 25 cents each.

7. What must be paid for 6 bales of cotton containing 20 pounds each, at 16⅔ cents a pound?

8. What will 18 pieces of calico, each containing 45 ards, cost at 25 cents a yard? *Ans.* $202.50.

CASE II.

130. To find the cost when the quantity and the price of 100 or 1000 are given.

1. What is the cost of 8409 feet of boards, at $28.25 a thous[illegible]

45

SOLUTION.—If 1000 feet cost $28.25, 1 foot will cost $\frac{1}{1000}$ of $28.25, and 8409 feet will cost 8409 times $\frac{1}{1000}$ of $28.25, which, by multiplying and cutting off three places in the product, we find to be $237.55.

WRITTEN EXERCISES.

is the cost:

Of 897 apples at $2.75 per hundred?

Of 84728 bricks at $9.50 per 1000?

4. Of 867 pounds of pork at $6.25 per hundred?

5. Of 94520 feet of stone at $48 a thousand?

6. Of 912 pounds of beef at $9 a hundred?

7. Of 6450 oysters at $3 a hundred? *Ans.* $193.50.

8. What will 8783 feet of hemlock scantling cost at $4.62½ per M.? *Ans.* $40.6213.

9. What will a compositor receive for setting up a book of 430 pages of 784 ems each, if he receive 65 cents per thousand? *Ans.* $219.128.

10. What is the cost of 3816 pounds of wheat flour at $8.25 per C.? *Ans.* $314.82.

Case III.

131. To find the cost of articles sold by the ton of 2000 pounds.

1. At $15.25 per ton, what will be the cost of 12845 pounds of hay?

OPERATION.

$$\begin{array}{r} 2\ \underline{)\ \$15.25} \\ \$7.625 \\ \underline{12.845} \\ \$97.943125 \end{array}$$

SOLUTION.—Since 1 ton, or 2000 pounds, cost $15.25, ½ ton, or 1000 pounds, will cost $15.25 ÷ 2 = $7.625. If 1000 pounds cost $7.625, 1 pound will cost $\frac{1}{1000}$ of $7.625; and 12845 pounds will cost 12845 times $\frac{1}{1000}$ of $7.625, which is $97.94.

WRITTEN EXERCISES.

What is the cost:

2. Of 5847 pounds of iron at $82.50 a ton?

3. Of 6540 pounds of wool at $580 a ton?

4. Of 3496 pounds of coal at $9.62½ a ton?

5. Of 36847 pounds of meal at $50 a ton?

6. Of 8420 pounds of pig iron at $34½ a ton?

7. Of 27816 pounds of oats at $26.75 a ton?

A peddler sold 2915 pounds of rags at $23 a ton ; at did he receive? *Ans.* $33.5225.

BILLS.

132. A ***Bill*** is a written statement of the purchase or e of goods. It should mention the time, the place, the rties concerned, the price of each item, and the entir st.

Find the amount of each of the following statements:

1. A Student's Expenses.

Oct. 1,	'87,	Paid tuition	$20.00
" 4,	"	Bought Greek Grammar	1.25
" 6,	"	Bought French Reader	1.00
" 6,	"	Bought Elementary Algebra	.85
" 15,	"	Bought pair boots	6.50
" 24,	"	Paid for car fare	2.90

2. A Family's Expenses.

Jan. 8,	'86,	Paid cook in full	$30.00
" 10,	"	Bought steak	1.80
" 12,	"	Bought bread	.95
" 19,	"	Bought 3 bushels potatoes	2.25
" 25,	"	Bought 2 pounds Java coffee	.70
" 29,	"	Paid servants in full	45.50

3. A Farmer's Expenses.

Mar. 1,	'85,	Bought a horse	$125.00
" 4,	"	Paid balance due Mr. Brown	60.00
" 10,	"	Paid 3 years' insurance	84.00
" 12,	"	Paid carpenter	24.50
" 16,	"	Bought a saddle	14.00
" 20,	"	Bought 60 bushels wheat	90.00
" 25,	"	Bought 40 pounds soap	3.40
" 26,	"	Paid for groceries	27.35
" 28,	"	Paid for repairing wagon	10.50
" 30,	"	Bought 85 bushels corn	34.00

4. A Painter's Receipts.

Dec.	1,	'84,	Received from W. Clarke on account...	$90.50
“	2,	“	Received from T. Stowe on account....	125.00
“	5,	“	Sold 8 pounds putty....................	.52
“	8,	“	Sold 10 gallons turpentine.............	6.00
“	9,	“	Sold paints and oils....................	79.50
“	11,	“	Received 60 days' painting.............	210.00
“	14,	“	Received from J. Wilkins on account..	83.50
“	20,	“	Sold 120 pounds green paint...........	36.00
“	27,	“	Received 152 days' painting...........	532.00

5. A Merchant's Receipts.

June	1,	'86,	Sold 42 yards cloth.....................	$105.00
“	4,	“	Sold 75 yards muslin....................	12.00
“	5,	“	Sold 34 yards ribbon....................	17.00
“	8,	“	Received from T. Curran on account...	98.50
“	13,	“	Sold 10 yards cloth.....................	45.00
“	15,	“	Sold 5 dozen handkerchiefs............	19.80
“	18,	“	Sold 14 dozen linen collars.............	35.00
“	25,	“	Received from W. Ross on account.....	38.25

Find the balance of each of the following accounts.

1. Henry's Cash Account.

Sept.	1,	'87,	Received from mother..................	$5.00
“	2,	“	Bought 4 school books	2.75
“	3,	“	Bought necktie..........................	.45
“	5,	“	Received from mother.................	1.50
“	7,	“	Bought 1 penknife......................	65
“	12,	“	Bought 6 handkerchiefs................	1.50

2. A Laborer's Transactions.

May	2,	'87,	Cash on hand............................	$41.75
“	3,	“	Received for 1 month's work...........	29.50
“	4,	“	Paid 2 weeks' board....................	10.00
“	6,	“	Bought 1 pair shoes....................	3.50
“	8,	“	Bought a hat and linen coat............	4.25
“	15,	“	Lent a friend............................	15.00

3. A Young Man at College.

Nov. 1, '83,	Cash on hand	$45.00
" 2, "	Paid 2d quarter's tuition	12.50
" 3, "	Paid books and stationery	10.50
" 4, "	Received from home	15.00
" 5, "	Bought overcoat and hat	23.50
" 7, "	Sold 3 books	1.85
" 10, "	Paid 3 weeks' board	13.50
" 19, "	Received from home	10.00
" 25, "	Lost	3.25
" 29, "	Paid 1 week's board	4.50

4. Find the gain or loss on an investment including the following transactions in raising potatoes:

May 1, '86	Plowing and harrowing, 8 days	@ $2.00
" 9, "	Plowing drills, 1½ days	" 1.25
" 12, "	Manuring in drills, 4 days	" $.56
" 13, "	Spreading manure, 1½ days	" .75
" 15, "	Seed potatoes, 3½ barrels	" 3.25
" 16, "	Cutting and dropping seed, 2 days	" 1.75
" 18, "	Covering with light plow, 1½ days	" 1.25
" 22, "	Hoeing and levelling, 1¾ days	" 1.50
June 25, "	Weeding, 3 days	" 1.00
July 10, "	Digging, 5 days	" 1.25
" 12, "	Sold 120 bushels	" .80
Aug. 15, "	Took 35 bushels for private use	" .70
" 16, "	Digging, 7 days	" 1.25
" 24, "	Sold 90 bushels	" .60

5. A Grocer's Transactions—Find gain or loss.

April 2, '84,	Bought 50 pounds soap	$5.25
" 5, "	Bought 12 barrels flour	90.00
" 7, "	Sold 30 pounds soap	3.60
" 8, "	Bought butter and sugar	75.00
" 11, "	Sold 6 barrels flour	48.00
" 14, "	Bought 87 pounds sugar	13.05
" 16, "	Sold 37 pounds sugar	5.18
" 19, "	Received from J. Marsh on account	24.00
" 21. "	Paid store expenses	53.50

6. As a merchant I deal with a butcher. Find the result of the following transactions:

Mar. 1, '87	Bought of H. Rah 10 pounds beef.... @ $0.22		
" 3, "	Bought of him 16 pounds mutton ... " .16		
" 5, "	Sold him 25 yards muslin .... " .13		
" 12, "	Received from him cash, per bill......($3.25)		
" 15, "	Bought of him 45 pounds pork .... ..@ .12		
" 16, "	" " 12 " corned beef " .10		
" 19, "	Sold him 15 yards cloth .... " 1.75		
" 21, "	Bought of him 20 pounds beef........ " .25		
" 26, "	Sold him 12 yards cambric.......... " .16		
" 29, "	Paid him cash, per bill................. ($1.20)		
" 31, "	Received from him, cash on account.($20.00)		

Find the amount of each of the following:

1. ST. LOUIS, Sept. 11, 1880.

MR. J. F. TONER,

Bought of H. W. WILLIAMS.

◇A	48	yards blue broadcloth. . @ $3.20		
2168	50	" cassimere . . . . @ $1.30		
W	2	pieces black broadcloth, each 42 yards . .84. . @ $2.20		

Received payment by note at 30 days,

H. W. WILLIAMS.

PER G. WHITE.

2. PHILADELPHIA, Aug. 7, 1882.

MR. H. CONNOLLY,

Bought of W. BRODERICK & Co.

Aug. 3	12	gallons vinegar. . . . . @ 16c.		
" "	28	" molasses. . . . . @ 40c.		
" 6	30	" kerosene. . . . . @ 17c.		
" "	10	" olive oil . . . . @ 80c.		

Received payment,

W. BRODERICK & Co.

PER T. BURNS.

3. NEW YORK, April 11, 1889.

MR. J. C. REILLY,

Bought of R. SALMON.

2	gallons molasses.........@ 50c.				
12	lb. rice...............@ 9c.				
4	" coffee (Rio).......@ 22c.				
3	" tea...............@ 50c.				
1	" almonds.........@ 27c.				
4	" raisins..........@ 20c.				
1	bbl. XXX. flour..........@	6	25		

Received payment,

R. SALMON.

4. BALTIMORE, July 27, 1881.

MR. J. SHEEHAN,

Bought of J. MÜLLER & SON.

May 13	15	gallons port wine.....@ $1.10				
" "	30	" brandy........@ $2.25				
June 17	24	" sherry........@ $1.67				
July 3	8	dozen champagne.....@ $9.00				
" "	16	" ginger ale......@ $1.20				
		Cr.				
" 7		By cash....................	100	00		
" 12		" "	15	25		
" 14		" "	10	50		
		Balance due J. M. & Son.				

Received payment,

J. MÜLLER & SON.

5. NEW YORK, May 21, 1888.

MR. JOHN KELLY,

Bought of H. JACKSON.

April 10	120	barrels flour...........@ $6.75				
" 25	75	" " ....@ $7.50				

WRITTEN EXERCISES.

Make out bills, accounts, and invoices, in proper form, from the following statements:

1.

21 pounds dried apples at 9c.
47 pounds tea at 39c.
15 pounds coffee at 46c.
35 pounds barley at 7c.
28 pounds raisins at 12c.
19 pounds lard at 9c.

2.

16 pounds cheese at 8c.
31 pounds soap at 9c.
40 pounds sugar at 7c.
12 pounds butter at 25c.
26 pounds rice at 10c.
20 pounds prunes at 18c.

3.

160 bushels rye at $1.10.
212 bushels turnips at 50c.
234 bushels onions at 86c.
311 bushels wheat at $1.35.
452 bushels corn at 80c.
370 bushels potatoes at 75c.

4.

122 barrels flour at $8.25.
375 barrels apples at $2.75.
216 barrels cider at $8.25.
324 barrels vinegar at $7.00.
112 bushels pears at $1.25.
340 bushels peaches at $2.25.

5. Mr. Burke ordered, June 23, 1883, from Thomas Crogan, the following: 6 lb. cheese @ 14c.; 2 lb. butter @ 30c.; 3 lb. cooking butter @ 18c.; 4 doz. eggs @ 15c.; 3 lb. lard @ 13c.

6. J. B. Larkin, Philadelphia, sold to Mrs. L. Regan, July 10, 1883, the following articles: 4 lb. soda crackers @ 6c.; 2 lb. egg biscuit @ 10c.; 8 loaves bread @ 6c.; 4 mince pies @ 12c; 3 lb. sponge cake @ 25c; 4 lb. ginger snaps @ 8c.

7. Mr. Philip A. Harvey, Paterson, N. J., presented the following statement to G. B. Grant, April 2, 1882: 4 pair French calf gaiters @ $6.00; 2 pair cloth slippers @ $2.50; half soles and heels on boots @ $1.50; 2 pair children's shoes @ $1,50; 1 doz. shoe laces @ 20c.; 1 razor strop @ 15c.

8. August 21, 1883, Mr. John F. Sanders, of Hoboken, . J., purchased of B. J. Murray & Co., N. Y., 32 tons rge stove coal @ $7.50; 13 tons nut coal @ $8.00; 15 rds wood @ $5.50; 25 bbl. kindling wood @ $0.50. ake out bill for amount.

9. Make out an appropriate bill on the supposition that u sold a gentleman 4 articles from a hardware store.

10. D. & J. Sadlier, New York, bought of Sower, Potts Co., Philadelphia, the following articles, June 22, 1883: Quackenbos' Natural Philosophy @ $0.90; 12 Shaw's terature @ $1.00; 20 Robinson's Geometry @ $0.85; 16 binson's University Algebra @ $0.95. They paid $30 cash, and returned books to the amount of $20. Make t bill showing entire statement.

11. M. J. Fuchs & Co., of New York, purchased of B. Vanere, Chicago, February 16, 1882, the following: gal. alcohol @ 75c.; 20 gal. brandy (Cognac) @ $7.50; gal. Scotch gin @ $3.50; 35 gal. Holland gin @ $4.50; gal. Century whiskey @ $4.00. What was the amount the bill?

12. Chas. H. Schreiber, of Cincinnati, shipped to T. F. arkson, of Buffalo, May 31, 1880, the following: 12 les, 5272 lb., Mid. Upland cotton @ 18c.; 10 bales, 4870 , Mid. Gulf cotton @ 20c.; 18 tierces Carolina rice, 00 lb., @ 6c. Make out bill, receipted.

13. On October 16, 1883, Mr. F. J. Quirk sold to Mr. A. Quigley, of Memphis, the following articles: 8 lb. nphor @ 50c.; 8 oz. quinine @ $3; 6 lb. Epsom salts @ ; 4 oz. morphine @ $5; 7 lb. gum shellac @ 75c.; 5 lb. a ammonia @ 8c.; and gave a receipted bill for amount.

14. Make out a bill of ten purchases from a grocery re.

DENOMINATE NUMBERS.

133. A ***Denominate Number*** is a concrete number whose unit is arbitrary.

134. There are two sorts of units; viz., *natural* and *arbitrary*.

135. A ***Natural Unit*** is a unit furnished by the nature of the quantity measured. Thus, in a row of houses, the unit is a house.

136. An ***Arbitrary Unit*** is a unit which is determined by custom or law. Thus, in the length of a road, the unit is the yard.

137. These units or measures produce two sorts of quantity: *discrete* and *continuous*.

138. ***Discrete Quantities*** are those which are composed of similar but distinct objects; as, a regiment of soldiers, a row of trees.

139. ***Continuous Quantities*** are those which do not present distinct parts; as, a surface, a solid, the weight of a body.

140. A ***Compound Number*** is a concrete number composed of different units of the same kind of quantity.

Denominate Numbers treat of:

1. Value.
2. Weight.
3. Length.
4. Surface.
5. Volume.
6. Capacity.
7. Time.
8. Angles.

MEASURES OF VALUE.

141. The ***Value*** of a thing is its price estimated on a piece of metal of determined weight.

142. ***Money*** is the measure by which we estimate the value of things.

143. ***Currency*** is money used as a circulating medium.

UNITED STATES CURRENCY.

144. The ***Unit*** of United States currency is the gold dollar. It weights 25.8 grains.

TABLE.

10 mills (m.) = 1 cent .... c. or ct.
10 cents = 1 dime .... d.
10 dimes = 1 dollar .... $.
10 dollars = 1 eagle .... E.

NOTE.—The *gold* coins are the $1, $2½, $3, $5, $10, and $20 pieces; the *silver* coins are the 10c., 25c., 50c., and $1 pieces; the *nickel* coins are the 3c. and 5c. pieces; the 1c. piece is made of *bronze*.

ORAL EXERCISES.

1. How many mills in a cent? In 3c.? In 5c.?

2. How many cents in a dime? In 2 dimes? In 4 dimes? In 10 dimes? In 15 dimes? In 24 dimes?

3. How many mills in 10 cents? Cents in a dime? Mills in a dime? Cents in 8 dimes? Mills in 20 dimes?

4. How many cents in a dollar? In $6? In $8?

5. How many mills in 12c.? In 60c.? In 90c.?

6. How many mills in 100 cents? Cents in a dollar? Mills in a dollar? Mills in 60 cents?

7. How many dimes in a dollar? In $10? In $7?

8. How many dimes in $2.50? In $6.40? In $9.30?

9. How many cents in 90 mills? In 30 m.? In 70 m.?

10. How many cents and mills in 65 m.? In 93 m.?

11. How many dimes in 80c.? In 20c.? In 110c.?

12. How many dimes in 65c.? In 92c.? In 215c.?

13. How many dollars in an eagle? In a double-eagle? In 6 eagles? In 9 eagles? In 15 eagles?

14. How many cents in 8 dimes 9 cents? 6 dimes 2 cents? 7 dimes 3 cents? 10 dimes 4 cents?

15. How many cents in the silver coins, taking one of each? 5 of each? 10 of each?

16. How many dollars in the gold pieces, taking one of each? 3 of each? 6 of each?

ENGLISH MONEY.

145. ***English Money*** is the legal currency of Great Britain.

146. The ***Unit*** of English money is the pound sterling.

TABLE.

4 farthings (far.) = 1 penny .... d.
12 pence = 1 shilling .... s.
20 shillings = 1 pound .... £.

Also, 2 shillings = 1 florin; 5 shillings = 1 crown; and 21 shillings = 1 guinea.

NOTE.—The gold coin valued at one £, is called a *sovereign*. Its value in U. S. money is $4.8665. The *gold* coins of Great Britain are the sovereign, half-sovereign, and guinea; the *silver* coins are the crown, half-crown, florin, shilling, six-penny, and three-penny pieces; the *copper* coins are the penny, half-penny, and farthing.

ORAL EXERCISES.

1. How many farthings in a penny? In 6 pence? In 7 pence? In 11 pence? In 9 pence? In 12 pence?

2. How many farthings in 3d. 2 far.? In 4d. 3 far.?

3. How many pence in a shilling? In 4 shillings? In 3 shillings? In 7 shillings? In 11 shillings?

4. How many farthings in 12 pence? Pence in a shilling? Farthings in a shilling?

5. How many farthings in 3s.? In 10s.? In 15s.?

6. How many shillings in a £? In £7? In £13? In £25? In £15? In £20? In £40?

7. How many shillings in a crown? In 6 crowns? In 14 crowns? In 20 crowns? In 9 crowns?

8. How many pounds in 60 shillings? In 100s.? In 80s.? In 480s.? In 360s.? In 640s.?

9. How many pence in 16 far.? In 18 far.? In 36 far.?

10. How many shillings in 72d.? In 96d.? In 240d.?

11. What will 5 lb. of beef cost at 9d. per pound?

12. How many yards of cloth at 6s. a yard can be purchased for £3? For £6? For £11?

13. What cost 36 combs at 9d. apiece? At 3d.?

14. How many bats at 2s. each may be bought for 120d.? For 240d.? For 144d.? For 72d.?

REDUCTION DESCENDING.

1. Reduce £3 11s. 9d. 2 far. to farthings.

OPERATION.

£ s. d. far.
3 11 9 2
20
71s.
12
861d.
4
3446 far.

Solution.—Since one £ = 20s., £3 = 60s.; adding 11s. to this, we have 71s. Since 1s. = 12d., 71s. = 852d., which together with 9d. give 861d. And, since 1d. = 4 far., 861d. = 3444 far., which, plus 2 far., give 3446 far. Hence, £3 11s. 9d. 2 far. = **3446 far.**

WRITTEN EXERCISES.

Reduce:

2. £3 8s. 5d. to pence. *Ans.* 821d.
3. £10 6s. 11d. to pence. *Ans.* 2483d.
4. £57 10s. 3d. 2 far. to far. *Ans.* 55214 far.
5. £60 9s. 2d. to farthings. *Ans.* 58040 far.
6. £35 0s. 10d. to farthings. *Ans.* 33640 far.
7. £18 16s. to pence. *Ans.* 4512d.
8. £150 to farthings. *Ans.* 144000 far.
9. 6s. 11d. to pence. *Ans.* 83d.
10. £1 1s. 1d. 1 far. to farthings. *Ans.* 1013 far.
11. £55 to pence. *Ans.* 13200d.
12. £180 18s. to shillings. *Ans.* 3618s.
13. £2860 to shillings. *Ans.* 57200s.

REDUCTION ASCENDING.

1. Reduce 7115 farthings to higher denominations.

OPERATION.

4) 7115 far.
12) 1778 + 3 far.
20) 148 + 2d.
£7 + 8s.
£7 8s. 2d. 3 far.

SOLUTION.—Since 4 far. = 1d., in 7115 far. there are 1778d. and 3 far. Also, 12d. = 1s.; hence, in 1778d., there are 148s. and 2d. Again, as 20s. = £1, so in 148s. there are £7 and 8s.

WRITTEN EXERCISES.

Change:

2. 6000 farthings to pounds sterling. *Ans.* £6 5s.
3. 23088d. to pounds. *Ans.* £96 4s.
4. 1684 far. to shillings. *Ans.* 35s. 1d.
5. 8152d. to pounds. *Ans.* £33 19s. 4d.
6. 66778 far. to pounds. *Ans.* £69 11s. 2d. 2 far.

7. 23694d. to pounds. *Ans.* £98 14s. 6d.
8. 16860 far. to shillings.
9. 5068d. to crowns. *Ans.* 84 crowns 2s. 4d.
10. 9790 far. to pence.
11. 27210d. to sovereigns. *Ans.* 113 sov. 7s. 6d.
12. 59317s. to pounds. *Ans.* £2965 17s.
13. 1048521d. to pounds.
14. 816 farthings to shillings. *Ans.* 17s.
15. 522315d. to florins. *Ans.* 21763 florins 3d.

MEASURES OF WEIGHT.

147. ***Weight*** is the measure of the earth's attraction.

148. The ***Unit*** of weight is the Troy pound.

149. There are three measures of weight in general use; viz., Troy Weight, Apothecaries' Weight, and Avoirdupois Weight.

TROY WEIGHT.

150. ***Troy Weight*** is used in weighing precious metals, jewels, and also in philosophical experiments.

151. The ***Unit*** of Troy Weight is the pound of 5760 grains.

TABLE.

24 grains (gr.) = 1 pennyweight.................... pwt.
20 pennyweights = 1 ounce.......................... oz.
12 ounces = 1 pound.......................... lb.

ORAL EXERCISES.

1. How many grains in 1 pwt.? In 4 pwt.? In 6 pwt.? In 10 pwt.? In 20 pwt.? In 3 pwt.?

2. How many pwt. in an ounce? In 3 oz.? In 5 oz.?

3. How many ounces in a lb.? In 5 lb.? In 8 lb.? In 12 lb.? In 15 lb.? In 30 lb.? In 50 lb.?

4. How many lb. in 60 oz.? In 48 oz.? In 288 oz.? In 72 oz.? In 120 oz.? In 144 oz.? In 84 oz.?

5. How many ounces in 60 pwt.? In 480 pwt.? In 180 pwt.? In 220 pwt.?

6. How many pwt. in 72 gr.? In 120 gr.? In 240 gr.?

7. How many grains in 4 pwt. 3 gr.? In 10 pwt. 10 gr.?

8. How many pwt. in 4 oz. 2 pwt.? In 10 oz. 6 pwt.?

9. How many oz. in 3 lb. 4 oz.? In 12 lb. 6 oz.? In 15 lb. 9 oz.? 10 lb. 3 oz.? 20 lb. 5 oz.?

10. At 3 cents a grain what will 2 pwt. of gold cost?

11. How many watches of 2 oz. each can be made from 80 pwt. of gold? From 120 pwt.?

12. What will 2 lb. of silver cost at 6 cents a pwt.?

WRITTEN EXERCISES.

Reduce:

1. 3 lb. 10 oz. 6 pwt. 3 gr. to grains. *Ans.* 22227 gr.
2. 9 lb. 0 oz. 11 pwt. 10 gr. to grains. *Ans.* 52114 gr.
3. 6 oz. 12 pwt. 17 gr. to grains. *Ans.* 3185 gr.
4. 18 lb. 6 oz. 4 pwt. to pwt. *Ans.* 4444 pwt.
5. 200 lb. 9 oz. 0 pwt. to pennyweights.
6. 34 lb. 6 gr. to grains. *Ans.* 195846 gr.
7. 10 oz. 19 pwt. to grains. *Ans.* 5256 gr.
8. 207 lb. to ounces; to pwt.
9. 76 lb. 10 oz. 16 pwt. to pennyweights.
10. 19 lb. 5 oz. to grains. *Ans.* 111840 gr.
11. 23036 gr. to lb. *Ans.* 3 lb. 11 oz. 19 pwt. 20 gr.
12. 3996 pwt. to lb. *Ans.* 16 lb. 7 oz. 16 pwt.
13. 523769 gr. to lb. *Ans.* 90 lb. 11 oz. 3 pwt. 17 gr.
14. 8406 pwt. to lb. *Ans.* 35 lb. 6 pwt.
15. 6004 gr. to ounces. *Ans.* 12 oz. 10 pwt. 4 gr.

APOTHECARIES' WEIGHT.

152. ***Apothecaries' Weight*** is used in prescribing and in compounding dry medicines.

153. The ***Unit*** of this measure is the Troy pound. This weight differs from Troy Weight only in the divisions of its ounce.

TABLE.

20 grains (gr.) = 1 scruple.................... scr., or ℈.
3 scruples = 1 dram....................... dr., or ʒ.
8 drams = 1 ounce....................... oz., or ℥.
12 ounces = 1 pound..................... lb., or ℔.

APOTHECARIES' LIQUID MEASURE.

Apothecaries' Liquid Measure is used in prescribing and in compounding liquid medicines.

60 drops (gtt.) or minims (♏) = 1 fluid dram........ fʒ.
8 fluid drams = 1 fluid ounce........ f℥.
16 fluid ounces = 1 pint.............. 0.
8 pints = 1 gallon............ Cong.

ORAL EXERCISES.

1. How many grains in a scruple? In 3 scr.? In 5 scr.? In 8 scr.? In 15 scr.? In 30 scr.?

2. How many scruples in 60 grains? In 70 gr.? In 80 gr.? In 40 gr.? In 55 gr.? In 95 gr.?

3. How many scruples in a dram? In 6 dr.? In 8 dr.? In 10 dr.? In 15 dr.? In 24 dr.?

4. How many drams in 6 scruples? In 24 scr.? In 15 scr.? In 36 scr.? In 51 scr.? In 72 scr.?

5. How many drams in an ounce? In 4 oz.? In 11 oz.? In 17 oz.? In 20 oz.? In 30 oz.?

6. How many pounds in 24 oz.? In 36 oz.? In 72 oz.?

7. How many grains in a dram? In 3 dr.? In 4 dr.?

8. How many drams in a lb.? In 3 lb.? In 7 lb.?

9. How many ounces in 3 lb. 2 oz.? In 2 lb. 4 oz.?

10. How many scruples in 3 dr. 2 scr.? In 12 dr. 2 scr.? In 10 dr. 1 scr.? In 15 dr. 4 scr.?

11. How many pills of 2 grains each, may be made from an ounce of quinine?

12. What is their value at 2 cents each?

WRITTEN EXERCISES.

Reduce:

1. 9 lb. 10 oz. 6 dr. 2 scr. to grains. *Ans.* 57040 gr.
2. 11 lb. ℥ 1 ʒ 6 to drams. *Ans.* ʒ 1070.
3. ℥ 16 ʒ 4 ℈2 to scruples. *Ans.* ℈398.
4. 1 lb. ℥ 1 ʒ 1 to grains. *Ans.* 6300 gr.
5. 16 lb. 2 oz. to scruples. *Ans.* ℈4656.
6. 3 oz. to grains. *Ans.* 1440 gr.
7. 46 lb. 10 oz. to grains. *Ans.* 269760 gr.
8. 270 lb. ℥ 9 to drams. *Ans.* ʒ 25992.
9. ʒ 7 ℈2 13 gr. to grains. *Ans.* 473 gr.
10. ℥ 11 ʒ 2 ℈1 16 gr. to grains. *Ans.* 5436 gr.
11. 57050 grains to lb. *Ans.* 9 lb ℥ 10 ʒ 6 ℈2 10 gr.
12. 8640 scruples to lb. *Ans.* 30 lb.
13. 34572 grains to lb. *Ans.* 6 lb. 12 gr.
14. 6000 grains to oz. *Ans.* 12 oz. 4 dr.
15. 78406 scruples to oz. *Ans.* ℥ 3266 ʒ 7 ℈1.
16. 8009 drams to lb. *Ans.* 83 lb. ℥ 5 ʒ 1.
17. 4040 grains to drams. *Ans.* ʒ 67 ℈1.
18. 16843 scruples to lb. *Ans.* 58 lb. 5 oz. 6 dr. 1 scr.
19. 4444 grains to oz. *Ans.* ℥ 9 ʒ 2 4 gr.
20. 862 drams to oz. *Ans.* ℥ 107 ʒ 6.

AVOIRDUPOIS WEIGHT.

154. ***Avoirdupois Weight*** is used in weighing all common goods.

155. The ***Unit*** of this measure is the pound of 7000 Troy grains.

TABLE.

16 ounces (oz.)	=	1 pound	lb.
100 pounds	=	1 hundredweight	cwt.
		or cental	C.
20 hundredweight	=	1 ton	T.

NOTE.—The Long ton contains 2240 pounds; 112 pounds to the hundredweight, and 28 pounds to the quarter. The Long ton is used at the U. S. Custom House, also in weighing ores, coal at the mines, and in ocean freights.

ORAL EXERCISES.

1. How many ounces in a pound? In 3 lb.? In 7 lb.? In 12 lb.? In 15 lb.? In 20 lb.? In 50 lb.?

2. How many pounds in 3 cwt.? In 6 cwt.? In 2 centals? In 6 C.? In 10 C.? In 35 C.? In 47 C.?

3. How many cwt. in a ton? In 3 tons? In 7 tons?

4. How many ounces in 2 lb.? In a cwt.? In a ton?

5. How many pounds in 80 oz.? In 32 oz.? In 96 oz.?

6. How many pounds and ounces in 37 oz.? In 46 oz.? In 89 oz.? In 75 oz.? In 65 oz.?

7. How many centals and pounds in 670 lb.? In 750 lb.? In 1010 lb.? In 2330 lb.?

8. What is the cost of a pig weighing 3 cwt. 50 lb., at 5 cents a lb.? At 6 cents? At 7 cents?

9. At 3 cents an ounce, what will 4 lb. of tea cost? 12 lb.? 30 lb.? 10 lb.?

10. What cost 4 centals of sugar, at 9 cents a lb.?

WRITTEN EXERCISES.

Reduce :

1. 7 T. 9 cwt. 16 lb. to lb. *Ans.* 14916 lb.

2. 1 T. 11 cwt. 70 lb. 15 oz. to oz. *Ans.* 50735 oz.

3. 6 cwt. 9 oz. to ounces. *Ans.* 9609 oz.

4. 8 T. 4 lb. to pounds. *Ans.* 16004 lb.

5. 17 T. 16 cwt. to cwt. *Ans.* 356 cwt.

6. 4 C. 6 lb. to oz. *Ans.* 6496 oz.

7. 25 T. 6 C. 84 lb. 12 oz. to oz. *Ans.* 810956 oz.

8. 25 C. 86 lb. to lb. *Ans.* 2586 lb.

9. 8 T. 2 C. to centals. *Ans.* 162 C.

10. 375 T. 16 C. 90 lb. 14 oz. to oz. *Ans.* 12027054 oz.

11. 470507 oz. to tons. *Ans.* 14 T. 14 cwt. 6 lb. 11 oz.

12. 82880 lb. to tons. *Ans.* 41 T. 8 cwt. 80 lb.

13. 43276 oz. to cwt. *Ans.* 27 cwt. 4 lb. 12 oz.

14. 6780 oz. to lb. *Ans.* 423 lb. 12 oz.

15. 30245 lb. to tons. *Ans.* 15 T. 2 C. 45 lb.

16. 438 cwt. to tons. *Ans.* 21 T. 18 cwt.

17. 16784 oz. to centals. *Ans.* 10 C. 49 lb.

18. 3700 oz. to centals. *Ans.* 2 C. 31 lb. 4 oz.

19. 18000 lb. to tons. *Ans.* 9 T.

20. 5214430 oz. to tons. *Ans.* 162 T. 19 C. 1 lb. 14 oz.

GENERAL WEIGHTS.

Commodities.	lb. per bu.	Commodities.	lb. per bu.	Commodities.	lb. per bu.
Apples (dry)...	28	Timothy seed ..	45	Potatoes.......	60
Peaches (dry)..	28	Salt	50	Pease..........	60
Oats	32	Rye...........	56	Beans..........	60
Malt..........	34	Corn (shelled)..	56	Wheat	60
Barley	48	Flax seed......	56	Clover seed ...	60
Buckwheat....	48	Onions	57	Corn (in ear)...	70

Commodities.	lb.	Commodities.	lb.
Barrel of flour	196	Quintal of fish.........	100
" pork.........	200	Keg of nails...........	100
" salt..........	280	Gallon of petroleum....	6½

ORAL EXERCISES.

1. How many lb. in a bushel of dried apples? In 6 bu.? In 11 bu.?

2. How many lb. in 2 bbl. of flour? In 3 kegs of nails? In 4 gallons of petroleum?

3. How many lb. in a bu. of buckwheat? In 3 bu. of potatoes? In 7 bu. of salt?

4. What cost 3 bbl. of pork at 9c. a lb.? At 11c. a lb.?

5. What cost 4 bu. of salt at 6c. a lb.? At 8c.?

6. Which is the heavier, a bushel of potatoes or a bushel of onions?

7. What cost 3 bu. of barley at 5c. a lb.? At 4c.?

8. What cost 4 quintals of codfish at 10c. a lb.? At 12c.?

WRITTEN EXERCISES.

1. A farmer sold 20 bushels of oats at 86c. a bu., and 16 bushels of buckwheat at 3c. a lb.; how much did he receive for both? *Ans.* $40.24.

2. What cost 9 bbl. of flour at 5c. a lb., 3 bbl. of salt at 4c. a lb., 4 bu. of dried peaches at 20c. a lb.?
Ans. $144.20.

3. What is the weight, in tons, of 3 loads of potatoes, averaging 22 bu. each; 1 load of wheat, 19 bu.; and 4 loads of oats, each 25 bu.? *Ans.* 4 T. 3 cwt.

4. Find the cost of 47½ quintals of fish at 5c. a pound, and 14⅞ gallons of petroleum at 16c. a gallon.

MEASURES OF LENGTH.

156. ***Measures of Length*** are those standards which enable us to estimate extent considered as a line; as, the *height* of a house, the *depth* of the sea.

157. The *Unit* of length is the yard. It is equal to $\frac{360000}{391392}$ of the length of a pendulum vibrating seconds, at London.

TABLE.

12 inches (in.) = 1 foot .... ft.
3 feet = 1 yard.............................. yd.
5½ yards, or 16½ feet = 1 rod.............................. rd.
320 rods = 1 mile .... m.

OTHER DENOMINATIONS.

12 lines = 1 inch.
3 sizes = 1 inch (shoemaker's).
4 inches = 1 hand (in measuring horses).
9 inches = 1 span.
6 feet = 1 fathom (depth at sea).
3 feet = 1 pace.
40 rods = 1 furlong.
8 furlongs = 1 mile.
1.152$\frac{2}{3}$ miles = 1 knot, or nautical mile.
60 geographical miles } = 1 degree of { latitude on a meridian.
69.16 statute miles } = 1 degree of { longitude on the Equator.

ORAL EXERCISES.

1. How many inches in a foot? In 3 ft.? In 5 ft.?

2. How many feet in 24 inches? In 72 in.? In 120 in.?

3. How many feet in a yard? In 2 yd.? In 6 yd.?

4. How many yards in 15 feet? In 21 ft.? In 33 ft.?

5. How many feet and inches in 37 inches? In 75 in.?

6. How many yards and feet in 20 feet? In 32 ft.?

7. How many yards in a rod? In 2 rd.? In 7 rd.?

8. How many rods in 11 yd.? In 22 yd.? In 44 yd.?

9. How many feet in a yard? Yards in a rod?
10. How many feet in 3 rods? In 6 rods? In 10 rd.?
11. How many inches in a foot? Feet in a yard?
12. How many inches in 6 yd.? In 10 yd.?
13. How many inches in 3 ft. 8 in.? In 10 ft. 6 in.?
14. How many feet in 3 yd. 2 ft.? In 4 yd. 1 ft.?
15. How many inches in 1 yd. 1 ft.? In 3 yd. 2 ft.?
16. How many feet in 3 rd. 2 yd.? 4 rd. 4 yd.?
17. How many rods in a furlong? In 3 furlongs?
18. How many furlongs in 80 rd.? In 120 rd.?
19. How many furlongs in a mile? In 6 m.? In 9 m.?
20. How many miles in 16 fur.? In 40 fur.? 72 fur.?
21. How many rods in a mile? In 6 m.? In 4 m.?
22. How many rods in 6 fur. 20 rd.? In 7 fur. 16 rd.?

WRITTEN EXERCISES.

Reduce:

1. 10 m. 303 rd. 4 yd. 2 ft. 11 in. to inches.
2. 37 rd. 3 yd. 7 in. to inches. *Ans.* 7441 in.
3. 69 m. 270 rd. 1 yd. to feet. *Ans.* 368778 ft.
4. 3 fur. 19 rd. 3 yd. 1 ft. to feet. *Ans.* 2303½ ft.
5. 4 yd. 2 ft. 11 in. to inches. *Ans.* 179 in.
6. 121 m. 280 rd. 5 yd. 9 in. to inches.
7. 3 fur. 28 rd. 4 yd. to yards. *Ans.* 818 yd.
8. 81 m. 3 fur. 38 rd. to rods. *Ans.* 26078 rd.
9. 95936 inches to miles.
10. 553248 in. to miles. *Ans.* 8 m. 234 rd. 1 yd.
11. 29876 ft. to miles. *Ans.* 5 m. 210 rd. 3 yd. 2 ft.
12. 215 inches to yd. *Ans.* 5 yd. 2 ft. 11 in.
13. 2640 feet to furlongs. *Ans.* 4 fur.
14. 37540 yards to miles.
15. 6552 inches to rods. *Ans.* 33 rd. 1 ft. 6 in.
16. 1754 yards to furlongs. *Ans.* 7 fur. 38 rd. 5 yd.

SURVEYORS' LINEAR MEASURE.

TABLE.

7.92 inches = 1 link .... l.
25 links = 1 rod .... rd.
4 rods or 100 links = 1 chain .... ch.
80 chains = 1 mile .... m.

ORAL EXERCISES.

1. How many inches in a link? In 2 l.? In 10 l.?
2. How many links in a rod? In 2 rd.? In 4 rd.?
3. How many rods in 75 links? In 125 l.? In 200 [illegible].?
4. How many rods in a chain? In 6 ch.? In 10 c[illegible].?
5. How many links in 3 ch.? In 7 ch.? In 20 ch.[illegible]
6. How many chains in 16 rd.? In 30 rd.? In 20 r[illegible].?
7. How many chains in 300 links? In 150 l.?
8. How many chains in a mile? In 3 m.? In 12 m[illegible].?
9. How many miles in 160 chains? In 400 ch.?
10. How many miles and chains in 250 ch.? In 90. c[illegible] l.?

WRITTEN EXERCISES.

Reduce:

1. 3 m. 70 ch. 43 l. to links. *Ans.* 31043 l.
2. 27 ch. 30 l. to links. *Ans.* 2730 l.
3. 13 m. 20 ch. 16 l. to inches. *Ans.* 839646.72 in.
4. 3 rd. 17 l. to links. *Ans.* 92 l.
5. 4 m. 60 ch. to links. *Ans.* 38000 l.
6. 26000 l. to miles. *Ans.* 3 m. 20 ch.
7. 3672 l. to chains. *Ans.* 36 ch. 2 rd. 22 l.
8. 4634 l. to rods. *Ans.* 185 rd. 9 l.
9. 35446 in. to chains. *Ans.* 44 ch. 75 l. 4 in.
10. 174191 l. to miles. *Ans.* 21 m. 61 ch. 91 l.

MEASURES OF SURFACE.

158. ***Measures of Surface*** are those measures by which we estimate extent considered under two dimensions; viz., length and breadth.

159. The ***Unit*** of Surface Measure is the square yard.

TABLE.

144	square inches (sq. in.)	= 1 square foot........	sq. ft.
9	square feet	= 1 square yard........	sq. yd.
30¼	square yards	= 1 square rod.........	sq. rd.
160	square rods	= 1 acre..............	A.
640	acres	= 1 square mile........	sq. m.

ORAL EXERCISES.

1. How many sq. in. in a sq. ft.? In 3 sq. ft.?

2. How many sq. ft. in 288 sq. in.? In 576 sq. in.?

3. How many sq. in. in 3 sq. ft. 72 sq. in.? In 2 sq. ft 19 sq. in.? In 5 sq. ft. 100 sq. in.?

4. How many sq. ft. in a sq. yd.? In 8 sq. yd.?

5. In 27 sq. ft. how many sq. yd.? In 45 sq. ft.? In 81 sq. ft.? In 72 sq. ft.? In 126 sq. ft.?

6. How many sq. ft. in 5 sq. yd. 6 sq. ft.? In 7 sq. yd. 8 sq. ft.? In 6 sq. yd. 10 sq. ft.?

7. How many sq. yd. in 4 sq. rd.? In 8 sq. rd.?

8. How many sq. rd. in an acre? In 4 A.? In 5 A.?

9. How many acres in 320 sq. rd.? In 800 sq. rd.?

10. How many acres in a sq. mile? In 2 sq. m.?

WRITTEN EXERCISES.

Reduce:

1. 3 sq. m. 500 A. 120 sq. rd. 20 sq. yd. 8 sq. ft. 79 sq in. to sq. in. *Ans.* 15184520431 sq. in.

2. 19 sq. yd. 6 sq. ft. 116 sq. in. to sq. in.

3. 620 A. 88 sq. rd. 25 sq. yd. to sq. yd.
Ans. 3003487 sq. yd.

4. 116 sq. rd. 30 sq. yd. 2 sq. ft. to sq. ft.

5. 18 sq. m. 210 A. 125 sq. rd. to sq. rods.
Ans. 1876925 sq. rd

6. 17 A. 20 sq. rd. 12 sq. yd. 7 sq. ft. to sq. ft.

7. 1 sq. m. 86 A. 10 sq. yd. to sq. ft.

8. 892 A. to sq. in.

9. 113941 sq. ft. to acres.
Ans. 2 A. 98 sq. rd. 15 sq. yd. 5 sq. ft. 72 sq. in.

10. 322704 sq. in. to sq. rd. *Ans.* 8 sq. rd. 7 sq. yd.

11. 6272640 sq. in. to acres. *Ans.* 1 A.

12. 8028979200 sq. in. to sq. miles. *Ans.* 2 sq. m.

13. 690871 sq. yd. to acres.
Ans. 142 A. 118 sq. rd. 21 sq. yd. 4 sq. ft. 72 sq. in.

14. 269407358 sq. in. to acres.

15. 6920 sq. ft. to sq. rd.

16. 29175 sq. in. to sq. yd.

SURVEYORS' SQUARE MEASURE.

TABLE.

625 square links = 1 square rod................ sq. rd.
16 square rods } = 1 square chain.............. sq. ch.
10000 square links }
10 square chains = 1 acre...................... A.
640 acres = 1 square mile................ sq. m.

WRITTEN EXERCISES.

Reduce :

1. 3 sq. m. 120 A. 6 sq. ch. to square links.
Ans. 204060000 sq. l.

2. 300 A. 80 sq. rd. to square links.
Ans. 30050000 sq. l.

3. 20 A. 7 sq. ch. 750 sq. l. to square links.
Ans. 2070750 sq. l.

4. 6 sq. m. 80 sq. ch. 12 sq. rd. to sq. rd.
Ans. 615692 sq. rd.

5. 7386407000 sq. links to square miles.
Ans. 115 sq. m. 264 A. 11 sq. rd. 125 sq. l.

6. 8600300 sq. links to acres. *Ans.* 86 A. 300 sq. l.

7. 36400 sq. rods to acres. *Ans.* 227 A. 5 sq. ch.

8. 240000 sq. links to square chains. *Ans.* 24 sq. ch.

MEASURES OF VOLUME.

160. ***Measures of Volume*** are those measures which serve to estimate extent considered under three dimensions; viz., length, breadth, and thickness.

TABLE.

1728 cubic inches (cu. in.) = 1 cubic foot........... cu. ft.
27 cubic feet = 1 cubic yard.......... cu. yd.

WOOD MEASURE.

16 cubic feet = 1 cord foot......................... cd. ft.
8 cord feet = 1 cord.............................. C.

NOTE.—Lumber is measured by board measure, whose unit is a board foot, 1 ft. long, 1 ft. wide, and 1 in. thick, or $\frac{1}{12}$ of a cubic foot.

All kinds of sawed timber, boards, planks, and joists are generally measured by this measure. But hewn and round timber are measured by cubic measure.

ORAL EXERCISES.

1. How many cu. ft. in a cu. yard? In 3 cu. yd.?

2. How many cu. yd. in 54 cu. ft.? In 81 cu. ft.? In 135 cu. ft.?

3. How many cu. ft. in 3 cu. yd. and 8 cu. ft.? In 6 cu. yd. and 7 cu. ft.? In 4 cu. yd. and 3 cu. ft.?

4. How many cu. ft. in a cd. ft.? In 3 cd. ft.? In 18 cd. ft.? In 11 cd. ft.?

5. How many cd. ft. in a cord? In 3 C.? In 8 C.? In 12 C.? In 10 C.? In 25 C.?

6. How many cu. ft. in 5 cords? In 2 C.? In 6 C.? In 9 C.? In 10 C.? In 20 C.?

WRITTEN EXERCISES.

Reduce :

1. 18 cu. yd. 3 cu. ft. 1600 cu. in. to cu. in. *Ans.* 846592 cu. in.

2. 218 cu. yd. 18 cu. ft. to cu. ft. *Ans.* 5904 cu. ft.

3. 13 cu. ft. 1692 cu. in. to cu. in. *Ans.* 24156 cu. in.

4. 8 C. 3 cd. ft. 2 cu. ft. to cu. ft. *Ans.* 1074 cu. ft.

5. 596 cu. yd. to cu. in. *Ans.* 27806976 cu. in.

6. 9 cords to cu. inches. *Ans.* 1990656 cu. in.

7. 921024 cu. in. to cu. yd. *Ans.* 19 cu. yd. 20 cu. ft.

8. 44850 cu. in. to cd. feet.

9. 2216 cu. ft. to cords.
Ans. 17 C. 2 cd. ft. 8 cu. ft., or 17 C. 40 cu. ft.

10. 2468784 cu. in. to cords.

MEASURES OF CAPACITY.

161. ***Measures of Capacity*** are those measures employed to measure liquids and dry matters.

LIQUID MEASURE.

162. ***Liquid Measure*** is used in measuring wine, molasses, etc.

163. The ***Unit*** of Liquid Measure is the gallon. It contains 231 cubic inches.

TABLE.

4 gills (gi.) = 1 pint.................................. pt.
2 pints = 1 quart.................................. qt.
4 quarts = 1 gallon .... gal.
31½ gallons = 1 barrel.................................. bbl.
63 gallons = 1 hogshead.................................. hhd.

OTHER DENOMINATIONS.

36 gallons = 1 barrel of beer.
42 gallons = 1 tierce.
54 gallons = 1 hogshead of beer.

ORAL EXERCISES.

1. How many gills in a pint? In 2 pt.? In 6 pt.? In 15 pt.? In 25 pt.?

2. How many pints in 12 gills? In 16 gi.? In 32 gi.?

3. How many gills in 1 pt. 2 gi.? In 2 pt. 3 gi.? In 4 pt. 1 gi.? In 7 pt. 3 gi.?

4. How many pints in a quart? In 10 qt.? In 15 qt.? In 45 qt.? In 18 qt.?

5. How many qt. in 24 pt.? In 80 pt.? In 72 pt.? In 34 pt.? In 18 pt.?

6. How many pt. in 4 qt. 1 pt.? In 7 qt. 1 pt.?

7. How many qt. and pt. in 87 pt.? In 49 pt.? In 33 pt.? In 17 pt.?

8. How many gills in a quart? In 3 qt.? In 4 qt.? In 11 qt.? In 17 qt.?

9. How many quarts in a gallon? In 15 gal.? In 3 gal.? In 11 gal.?

10. How many quarts in 10 gal. 3 qt.? In 13 gal. 2 qt.? In 27 gal. 1 qt.?

11. How many gal. and qt. in 83 qt.? In 25 qt.? In 33 qt.? In 75 qt.?

12. How many gallons in a barrel? In 2 bbl.? In 6 bbl.? In 8 bbl.? In 10 bbl.?

13. How many gallons in a hhd.? In 2 hhd.? In 4 hhd.? In 10 hhd.?

14. How many hhd. in 126 gal.? In 315 gal.?

15. How many pints in a gal.? In 3 gal.? In 17 gal.? In 20 gal.? In 25 gal.?

16. How many qt. in a hhd.? In 4 hhd.? In 10 hhd.? In 12 hhd.? In 8 hhd.? In 20 hhd.?

17. How many hhd. and gal. in 150 gal.? In 180 gal.? In 240 gal.? In 120 gal.? In 250 gal.?

18. How many gal. in 3 hhd. 40 gal.? In 12 hhd. 1 gal.? In 10 hhd. 50 gal.?

WRITTEN EXERCISES.

Reduce:

1. 6 gal. 3 qt. 1 pt. 3 gi. to gills. *Ans.* 223 gi.
2. 3 hhd. 60 gal. 3 qt. to pints. *Ans.* 1998 pt.
3. 18 hhd. 37 gal. 2 qt. to quarts. *Ans.* 4686 qt.
4. 2 bbl. 30 gal. 2 qt. 1 pt. to pints. *Ans.* 749 pt.
5. 30 bbl. to pints. *Ans.* 7560 pt.
6. 9 gal. 3 gi. to gills. *Ans.* 291 gi.
7. 16 hhd. to pints. *Ans.* 8064 pt.
8. 128 gal. to pints. *Ans.* 1024 pt.
9. 158 gi. to gallons. *Ans.* 4 gal. 3 qt. 1 pt. 2 gi.
10. 4039 pt. to hhd. *Ans.* 8 hhd. 3 qt. 1 pt.
11. 3310 qt. to hhd. *Ans.* 13 hhd. 8 gal. 2 qt.
12. 38609 gi. to gallons. *Ans.* 1206 gal. 2 qt. 1 gi.
13. 6780 pt. to hhd. *Ans.* 13 hhd. 28 gal. 2 qt.
14. 8080 pt. to bbl. *Ans.* 32 bbl. 2 gal.
15. 7824 gal. to hhd. *Ans.* 124 hhd. 12 gal.
16. 372 pt. to gallons. *Ans.* 46 gal. 2 qt.

DRY MEASURE.

164. ***Dry Measure*** is used in measuring articles not liquid; as grain, salt, vegetables, etc.

165. The ***Unit*** of Dry Measure is the bushel. It contains 2150.42 cubic inches.

TABLE.

2 pints (pt.) = 1 quart.. qt.
8 quarts = 1 peck.. pk.
4 pecks = 1 bushel.. bu.

ORAL EXERCISES.

1. How many pints in a quart? In 2 qt.? In 6 qt.?
2. How many qt. in 20 pints? In 14 pt.? In 26 pt.?
3. How many qt. and pt. in 17 pt.? In 31 pt.?
4. How many pt. in 3 qt. 1 pt.? In 19 qt. 1 pt.?
5. How many qt. in a peck? In 9 pk.? In 13 pk.?
6. How many pecks in 8 qt.? In 72 qt.? In 36 qt.?
7. How many pints in a peck? In 3 pk.? In 7 pk.?
8. How many pk. in 16 pt.? In 64 pt.? In 128 pt.?
9. How many pk. and qt. in 30 qt.? In 45 qt.?
10. How many pecks in a bushel? In 3 bu.? In 4 bu.?
11. How many bushels in 40 pk.? In 48 pk.? In 68 pk.?
12. How many quarts in a bushel? In 2 bu.? In 3 bu.?

WRITTEN EXERCISES.

Reduce:

1. 16 bu. 3 pk. 2 qt. 1 pt. to pints. *Ans.* 1077 pt.
2. 39 bu. 7 qt. to quarts. *Ans.* 1255 qt.
3. 8 bu. 3 pk. 1 pt. to pints. *Ans.* 561 pt.
4. 7 pk. 7 qt. to pints. *Ans.* 126 pt.
5. 12 bu. 3 pk. to quarts. *Ans.* 408 qt.
6. 1806 pt. to bushels. *Ans.* 28 bu. 7 qt.

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7. 8679 qt. to bushels. *Ans.* 271 bu. 7 qt.

8. 176 pt. to pecks. *Ans.* 11 pk., or 2 bu. 3 pk.

9. 18090 qt. to bushels. *Ans.* 565 bu. 1 pk. 2 qt.

10. 2176 pt. to pecks, then to bushels.
Ans. 136 pk.; 34 bu.

MEASURES OF TIME.

166. ***Measures of Time*** are those used to measure periods of duration.

167. The ***Unit*** of this measure is the day.

TABLE.

60 seconds (sec.)	= 1 minute	min.
60 minutes	= 1 hour	hr.
24 hours	= 1 day	da.
7 days	= 1 week	wk.
4 weeks	= 1 lunar month	l. mo.
365 days	= 1 common year	yr.
366 days	= 1 leap year	l. yr.
100 years	= 1 century	C.

NOTE.—All years divisible by 4, except centennial, and all centennial years divisible by 400 are leap years.

OTHER DENOMINATIONS.

52 weeks 1 day = 1 year.
13 lunar months, 1 day, 6 hours, = 1 year.
12 calendar months = 1 year.

CALENDAR.

1.	January (Jan.)	31 days.		7.	July	31 days.
2.	February (Feb.),	28 or 29 "		8.	August (Aug.)	31 "
3.	March (Mar.)	31 "		9.	September (Sept.)	30 "
4.	April (Apr.)	30 "		10.	October (Oct.)	31 "
5.	May	31 "		11.	November (Nov.)	30 "
6.	June	30 "		12.	December (Dec.)	31 "

ORAL EXERCISES.

1. How many seconds in a minute? In 5 min.? In 8 min.? In 10 min.? In 50 min.? In 40 min.?

2. How many minutes and seconds in 70 sec.? In 90 sec.? In 68 sec.? In 110 sec.?

3. How many seconds in 3 min. 5 sec.? 4 min. 30 sec.?

4. How many minutes in an hour? In 2 hr.? In 5 hr.?

5. How many hours and minutes in 90 min.? In 110 min.? In 85 min.? In 125 min.? In 170 min.?

6. How many hours in a day? In 5 days? In 7 days?

7. How many days in 48 hr.? In 120 hr.? In 96 hr.?

8. How many days in a week? In 30 wk.? In 70 wk.?

9. How many weeks in 14 days? In 56 da.? In 70 da.?

10. How many weeks in a month? In 3 mo.? In 7 mo.?

11. How many months in a year? In 3 yr.? In 9 yr.?

12. How many years in 48 mo.? In 60 mo.? In 132 mo.?

13. How many years and months in 20 mo.? In 37 mo.?

14. How many days in February? In June? In April?

15. How many days in January? In September? In March? In August?

16. How many days in October? In July? In November? In December?

17. How many months have 30 days? Name them.

18. What month has less than 30 days?

19. What months have 31 days?

WRITTEN EXEXCISES.

Reduce:

1. 4 da. 7 hr. 20 min. 50 sec. to sec. *Ans.* 372050 sec.

2. 11 da. 31 min. 59 sec. to seconds. *Ans.* 952319 sec.

3. 11 hr. 10 min. 9 sec. to seconds. *Ans.* 40209 sec.

4. 1 yr. 1 da. 1 hr. to minutes. *Ans.* 527100 min.

5. 8 yr. 160 da. 7 hr. to hours.
Ans. Counting two leap years, 73975 hr.

6. 4 wk. 3 da. 1 hr. 10 min. to min. *Ans.* 44710 min

7. How many seconds in March? *Ans.* 2678400 sec

8. How many minutes in May? *Ans.* 44640 min

9. How many hours in June? *Ans.* 720 hr

Reduce:

10. 86400 seconds to days. *Ans.* 1 da.

11. 3780 minutes to days. *Ans.* 2 da. 15 hr.

12. 18080 hr. to years. *Ans.* 2 yr. 23 da. 8 hr.

13. 78000 seconds to hours. *Ans.* 21 hr. 40 min.

14. 1208 minutes to hours. *Ans.* 20 hr. 8 min.

15. 186078 hours to years. *Ans.* 21 yr. 88 da. 6 hr.

16. Find the number of days from March 21st to May 16th.

OPERATION.

31 da. − 21 da. = 10 da. Mar.
30 da. April.
16 da. May.
Ans. 56 da.

SOLUTION.—Since March has 31 da., and the given date is March 21st, there are ten days to the end of the month. The intervening month, April, has 30 da., which are added to the 10 da. of Mar. The second given date is May 16th; hence, 16 da. more are added to the 10 da. + 30 da., making 56 days, the number of days from March 21st to May 16th.

How many days from:

17. January 17th to March 13th? *Ans.* 55 da.

18. February 16th to May 1st (com. yr.)? *Ans.* 74 da.

19. July 4th to Sept. 3d? *Ans.* 61 da.

20. August 8th to December 23d? *Ans.* 137 da.

21. April 1st to October 27th? *Ans.* 209 da.

22. January 11th to February 9th? *Ans.* 29 da.

23. September 30th to Nov. 30th? *Ans.* 61 da.
24. March 26th to May 3d? *Ans.* 38 da.
25. What is the date for 50 days after March 7th?

OPERATION.

31 da. − 7 da. = 24 da.
50 da. − 24 da. = 26 da.
April 26th.

SOLUTION. — Since from March 7th to March 31st there are 24 da., the date will be 50 da. − 24 da., or 26 days after Mar. 31, which is April 26th.

What is the date for:

26. 63 days after May 4th? *Ans.* July 6th.
27. 45 days after April 19th? *Ans.* June 3d.
28. 16 days after September 28th? *Ans.* Oct. 14th.
29. 33 days after January 26th? *Ans.* Feb. 28th.
30. 210 days after March 17th? *Ans.* Oct. 13th.
31. 69 days after August 11th? *Ans.* Oct. 19th.
32. What date is 87 days before June 6th?

OPERATION.

87 da. − 6 da. = 81 da.
81 da. − 31 da. = 50 da.
50 da. − 30 da. = 20 da.
31 da. − 20 da. = 11 da.

SOLUTION. — Subtracting 6 da. from 87 da. we have 81 da., the number of days before May 31st. Taking 31 da. we have 50 da., the number of days before April 30th. Subtracting 30 da. we have 20 da., the number of days before March 31st. Hence, 87 da. before June 6th is March 11th.

What date is:

33. 47 days before March 3d? *Ans.* Jan. 15th.
34. 83 days before May 17th? *Ans.* Feb. 23d.
35. 93 days before July 6th? *Ans.* April 4th.
36. 60 days before April 19th? *Ans.* Feb. 18th.
37. 180 days before August 1st? *Ans.* Feb. 2d.
38. 308 days before December 21st? *Ans.* Feb. 16th.
39. 201 days before October 11th?
40. 252 days before September 10th?

CIRCULAR MEASURE.

168. ***Circular Measure*** is used to measure angles.

169. The ***Unit*** of Circular Measure is the degree. It is the $\frac{1}{360}$ of the circumference of any circle.

TABLE.

60 seconds (″) = 1 minute .... ′
60 minutes = 1 degree.............................. °
360 degrees = 1 circumference...................... C.

OTHER DENOMINATIONS.

30 degrees = 1 sign.
60 degrees = 1 sextant.
90 degrees = 1 quadrant.
4 quadrants = 1 circumference.
12 signs = 1 circumference.

ORAL EXERCISES.

1. How many seconds in a minute? In 3′? In 5′?

2. How many minutes in a degree? In 2°? In 4°?

3. How many minutes in 300″? In 120″? In 480″?

4. How many degrees in 180′? In 300′? In 600′?

5. How many degrees in a sign? In 7 signs? In 11 signs? In 15 signs? In 40 signs?

6. How many degrees in a sextant? In 2 sextants?

7. How many degrees in a quadrant? In 3 quad.?

8. How many quadrants and degrees in 170°? In 250°? In 310°? In 205°?

9. How many signs in 60 degrees? In 90 degrees? In 45 degrees? In 70 degrees? In 140 degrees?

10. How many quadrants in a circumference? In 5 circumferences?

WRITTEN EXERCISES.

Reduce:

1. 13° 23′ 45″ to seconds. *Ans.* 48225″.

2. 180° 16″ to seconds. *Ans.* 648016″.

3. 1 quad. 45° 30″ to seconds. *Ans.* 486030″.

4. 270° 45′ to minutes. *Ans.* 16245′.

5. 2° 20′ 22′ to seconds. *Ans.* 8422″.

6. 3 signs to minutes. *Ans.* 5400′.

7. 15° 29′ 37″ to seconds. *Ans.* 55777″.

8. 1963000″ to higher denominations.

9. 96000″ to degrees. *Ans.* 26° 40′.

10. 38000″ to higher denominations.

11. 86078″ to degrees. *Ans.* 23° 54′ 38″.

12. 1897000″ to circumferences.

MISCELLANEOUS TABLES.

COUNTING.

12 units = 1 dozen.
12 dozen = 1 gross.
12 gross = 1 great gross.
20 units = 1 score.

PAPER.

24 sheets = 1 quire.
20 quires = 1 ream.
2 reams = 1 bundle.
5 bundles = 1 bale.

ORAL EXERCISES.

1. How many articles in a dozen? In 3 dozen? In 5 dozen? In 13 dozen?

2. How many articles in a ½ dozen? In 3½ dozen? In 7½ dozen? In 6½ dozen? In 12 dozen?

3. How many dozen in a gross? In 6 gross? In 15 gross? In 30 gross? In 25 gross?

4. How many gross in 60 dozen? In 48 dozen? In 144 dozen? In 156 dozen? In 288 dozen?

5. How many score in 80? In 60? In 140? In 200? In 75? In 125? In 120?

6. How many years in 4 score 10? In 3 score 5?

7. How many sheets in a quire? In 3 quires? In 7 quires? In 2 quires? In 24 quires?

8. How much paper in 48 sheets? In 96 sheets? In 70 sheets? In 60 sheets?

9. How many quires in a ream? In 30 reams? In 16 reams? In 20 reams? In 25 reams?

10. What cost a ream of paper at 2 cents a sheet?

WRITTEN EXERCISES.

1. How many pencils in a box containing 2 great gross? *Ans.* 3456 pencils.

2. What cost 27 boxes of writing ink, each containing 2½ dozen bottles, at 9 cents a bottle? *Ans.* $72.90.

3. How many reams of paper in 4678 sheets?

4. What will 7 reams of legal cap cost at 35 cents a quire? *Ans.* $49.

5. What cost 9 boxes of fancy pen-holders, each containing ½ gross, at 2½ cents apiece? *Ans.* $16.20.

* * *

MENTAL REVIEW.

1. What cost 2 oz. of gold, if 3 pwt. cost $2.70?

2. What will 2 quarts of kerosene cost at 40 cents a gallon? At 20 cents a gallon? At 50 cents a gallon?

3. What will 3 quarts of tomatoes cost at $1.60 a bushel? 10 quarts? 25 quarts?

4. How many feet high is a horse 16 hands high?

5. What is the difference between two square feet and two feet square?

6. At 8 cents a peck, how many bushels of apples can be bought for $6.00?

7. If 25 lb. of flour cost $1.25, what will 2 cwt. cost?

8. How many half-pint bottles may be filled from 2½ gallons of wine?

9. What will 7 quires of paper cost at $3.20 a ream?

10. What will 8 eggs cost at 18 cents a dozen?

11. If 6 oz. of tea cost 36 cents, what will 3 lb. cost?

12. What will a gallon of molasses cost at 5 cents a pint? At 8 cents a pint? At 15 cents a pint?

13. At 8 shillings a pair, how many pairs of shoes can be purchased for 2 sovereigns?

14. At what price must ⅓ dozen of chairs, worth $15.00 a dozen, be sold in order to gain 50 cents apiece?

15. How much will a peddler gain by selling 3 dozen combs worth 30 cents a dozen, at 5 cents apiece?

16. What will ⅝ of a lb. of candy cost at 2 cents an oz.? ¾ of a lb.? ½ of a lb.?

17. How many tablespoons each weighing 2 oz., can be made from 1 lb. 8 oz. of silver?

18. How many leap years in a century?

19. How many pills of 5 grains each, can be made from ½ an ounce of quinine?

20. If a gallon of wine cost $5.00, what will 3 pt. cost? ½ of a pint? 3 quarts?

21. What will it cost to paint a ceiling 12 ft. by 20 ft., at 25 cents a square yard?

22. How many yards of carpeting, a yard wide, will cover a floor 20 ft. long and 21 ft. wide?

23. How many quarts of milk will a boy drink in a week, if he drink a pint a day?

24. At 5 cents a quart, how many bushels of chestnuts can I get for $3.20?

25. If the wages of 8 men for 5 days is $90, what does each man receive per day?

WRITTEN REVIEW.

1. What will it cost to ship 75 T. 8 cwt. 70 lb. of freight, at 6 cents a pound? *Ans.* $9052.20.

2. How many farms of 75 acres each, in a tract of land 6 miles long and 5 miles wide? *Ans.* 256 farms.

3. What is the height in feet of a horse 17½ hands high? *Ans.* 5 ft. 10 in.

4. What will be the cost of a cask of sugar weighing 9 cwt. 68 lb., at 6¼ cents a pound? *Ans.* $60.50.

5. A dozen silver tablespoons weighed 2 lb. 2 oz.; what did they cost at $2.75 per ounce? *Ans.* $104.50.

6. How much will 11 barrels of flour cost, at the rate of 5¼ cents a pound? *Ans.* $113.19.

7. If your age be 15 years and 7 months, how many minutes old are you, counting 3 leap years, and supposing you were born on Jan. 1st?

8. What cost 13 bushels of clover seed at 13½ cents a pound? *Ans.* $105.30.

9. How many minutes in the summer months?

10. What will 5 bushels of potatoes cost at 15 cents per half-peck? *Ans.* $6.00.

11. How many times will a wheel 8 ft. 3 inches in circumference, revolve in going 45 miles? *Ans.* 28800 times.

12. How many half-pint, pint, and quart bottles, of each an equal number, can be filled from 2 barrels of wine?

13. How many rods of fence will inclose a farm a mile square? *Ans.* 1280 rods.

14. In what time will a person walk 8 miles, if he take a step 2 feet 8 inches long every second? *Ans.* 4 hr. 24 min.

15. How many yards of carpeting $1\frac{1}{2}$ yards wide, will be required to carpet a room 35 feet long, by 16 feet wide?
Ans. $41\frac{13}{27}$ yards.

16. In a pile of wood 50 feet long, 18 feet high, and 6 feet deep, how many cords? *Ans.* $42\frac{3}{16}$ cords.

17. What cost 3 T. 6 cwt. 78 lb. of iron at 11 cents a pound? *Ans.* $734.58.

18. A barrel of vinegar containing $31\frac{1}{2}$ gallons, was retailed at 12 cents a quart. How much was received for it? *Ans.* $15.12.

19. How much time will a person gain in 35 years by rising 35 minutes earlier than his usual time every day?
Ans. 310 da. 16 hr. 45 m.

20. A farmer sold a load of corn weighing 2296 lb. at 56 cents a bushel, and a load of wheat weighing 2400 lb. at $1.15 per bushel. What did he receive for both?
Ans. $68.96.

21. How long will it take a man to dig a plot of 2 acres, if he dig 80 sq. yd. a day? *Ans.* 121 da.

22. What will it cost to carpet a room 18 feet long by 16 feet wide, at $1.25 per sq. yd? *Ans.* $40.

23. A pile of wood is 8 feet high, 4 feet deep, and 188 feet long; what is its value at $6.50 a cord?

24. During a storm at sea a ship changed her course 280 geographical miles; how many degrees and minutes did she change? *Ans.* 4° 40′.

25. What will be the cost of plastering a room 24 feet long, 16 feet wide, and 12 feet high, at 25 cents a square yard, allowing 84 sq. ft. for windows and a door?
Ans. $35.

26. How many days of 12 working hours each, will it take a person to count 1600000, at the rate of 80 in a minute? *Ans.* 27 da. 9 hr. 20 min.

27. How many demijohns containing 1 gal. 1 qt. 2 gi. can be filled from a hhd. of wine?

28. What will it cost to dig a cellar 25 feet long, 19 feet wide, and 7 feet deep at 23 cents a cu. yd.?

29. A grocer sold 4 hhd. of syrup, containing 60 gal. 1 qt. each, at 18 cents a quart. What did he gain dishonestly, if his quart measure was too small by one gill?
Ans. $24.48½.

30. What will be the cost of excavating a ditch 4 miles long, with an average depth of 6 feet, and width of 4 feet, at 40 cents a cubic yard? *Ans.* $7509.33⅓.

31. A boy gathered 5 bu. 2 pk. of apples, which he sold at the rate of 5 for 12 cents. If each peck contained 2⅓ dozen apples, how much did he receive for the entire quantity? *Ans.* $14.78⅖.

32. If a brick measure 64 cubic inches, how many such bricks will it take to build a wall 60 feet long, 2 feet thick, and 12 feet high? *Ans.* 38880 bricks.

33. The front of a hall is 48½ feet wide and 24 feet high. What will it cost to paint it at the rate of 33 cents per square yard, allowing for 6 windows each 8 feet by 3 feet, and one door 14 ft. by 6 ft.? *Ans.* $34.32.

34. What will it cost to glaze 6 windows of 12 panes each, and each pane 14 inches by 10 inches; the glass being 35 cents per square foot? *Ans.* $24.50.

35. A garden 380 ft. long and 200 ft. wide is enclosed by a flagged walk 8 ft. in breadth. What is the area of the walk? *Ans.* 35 sq. rd. 7 sq. ft. 36 sq. in.

36. How many sods 16 in. by 9 in., will be required to turf a plot 64 ft. by 80 ft.? *Ans.* 5120 sods.

37. How many paving stones 8 in. by 5 in. will be required to pave a street 10 rods long by 60 feet wide?
Ans. 35640 stones.

38. What is a person's income for Jan., Feb., and Mar., at ~~the rate~~ of 3 cents a minute? *Ans.* $3888.

~~39~~. A man buys a barrel of beer (36 gal.) for $9.50, and retails it at 14 cents a quart. How much does he gain? 10.66

DENOMINATE FRACTIONS.

170. A *Denominate Fraction* is a fraction whose unit is denominate.

CASE I.

171. To reduce denominate fractions to integers of lower denominations.

ORAL EXERCISES.

1. How many ounces in $\frac{3}{4}$ of a pound avoirdupois?

SOLUTION.—Since in one pound there are 16 ounces, in $\frac{3}{4}$ of a pound there are $\frac{3}{4}$ of 16 ounces, or 12 ounces.

2. How many days in $\frac{3}{7}$ of a week? In $\frac{2}{3}$ of a week?

3. How many quarts in $\frac{7}{8}$ of a peck? $\frac{3}{4}$ of a peck?

4. How many quarts in $\frac{1}{2}$ of a gallon? In $\frac{1}{8}$ of a gallon?

5. How many pwt. in $\frac{3}{5}$ of an ounce? $\frac{1}{4}$ of an ounce?

6. How many cwt. in $\frac{3}{4}$ of a ton? $\frac{2}{5}$ of a ton? $\frac{3}{10}$ of a ton? $\frac{1}{4}$ of a ton? $\frac{4}{5}$ of a ton?

7. How many inches in .5 of a foot?

SOLUTION.—Since in 1 foot there are 12 inches, in .5 of a foot there are .5 of 12 inches, or 6 inches.

8. How many sq. rd. in .3 of an acre? .5 of an acre?

9. What is the value of .25 of an hour? .125 of an hour? .75 of an hour? $\frac{5}{12}$ of an hour?

10. How many pwt. and gr. in .3 of an oz.? In .7 of an oz.?

WRITTEN EXERCISES.

1. Reduce $\frac{3}{5}$ of a bushel to lower denominations.

OPERATION.

$\frac{3}{5}$ bu. $\times 4 = \frac{12}{5} = 2\frac{2}{5}$ pk.
$\frac{2}{5}$ pk. $\times 8 = \frac{16}{5} = 3\frac{1}{5}$ qt.
$\frac{1}{5}$ qt. $\times 2 = \frac{2}{5}$ pt.
$\frac{3}{5}$ bu. $= 2$ pk. 3 qt. $\frac{2}{5}$ pt.

SOLUTION.—Since 4 pk. make a bu., in $\frac{3}{5}$ of a bu. there are $\frac{3}{5} \times 4$ pk. $= \frac{12}{5}$ pk., or $2\frac{2}{5}$ pk. There being 8 qt. in a pk., $\frac{2}{5}$ of a pk. $= \frac{2}{5} \times$ 8 qt., or $3\frac{1}{5}$ qt. As there are 2 pt. in a qt., in $\frac{1}{5}$ of a qt. there is $\frac{1}{5}$ of 2 pt., or $\frac{2}{5}$ of a pt. Hence, $\frac{3}{5}$ of a bushel = 2 pk. 3 qt. $\frac{2}{5}$ pt., *Ans.*

2. Reduce .375 of a yard to lower denominations.

SOLUTION.—Since there are three feet in a yard, .375 of a yard equals .375 times 3 ft. = 1.125 ft. To reduce .125 ft. to inches, multiply it by 12: .125 ft. × 12 = 1.5 in. Hence, .375 yd. = 1 ft. 1½ in., *Ans.*

Reduce to lower denominations:

3. $\frac{5}{8}$ of a rod. *Ans.* 3 yd. 1 ft. $3\frac{3}{4}$ in.
4. $\frac{3}{7}$ of a degree. *Ans.* $25' \, 42\frac{6}{7}''$.
5. $\frac{8}{9}$ of a mo. *Ans.* 26 da. 16 hr.
6. $\frac{5}{6}$ of an A. *Ans.* 133 sq. rd. 10 sq. yd. 108 sq. in.
7. $\frac{1}{8}$ of a bbl. *Ans.* 3 gal. 3 qt. 1 pt. 2 gi.
8. $\frac{3}{5}$ of a cu. yd. *Ans.* 16 cu. ft. $345\frac{3}{5}$ cu. in.
9. $\frac{8}{9}$ of a lb. (Troy). *Ans.* 10 oz. 13 pwt. 8 gr.
10. $\frac{5}{6}$ of a lb. (Apoth.) *Ans.* 10 oz.
11. .214 of a T. *Ans.* 4 cwt. 28 lb.
12. .6 of a lb. (Troy). *Ans.* 7 oz. 4 pwt.
13. .3125 of a great gross. *Ans.* 45 doz.
14. .375 of a sign. *Ans.* 11° 15′.
15. .4267 of a bu. *Ans.* 1 pk. 5 qt. 1.308 pt.
16. $.66\frac{2}{3}$ of an A.
Ans. 106 sq. rd. 20 sq. yd. 1 sq. ft. 72 sq. in.
17. .625 of a yr. *Ans.* 228 da. 3 hr.

CASE II.

172. To reduce a compound number to a fraction of a higher denomination.

ORAL EXERCISES.

1. What fraction of a pound in 12 ounces (avoir.)?

SOLUTION.—Since there are 16 ounces in a pound, 12 ounces are $\frac{12}{16}$, or $\frac{3}{4}$ of a pound.

2. What part of an hour is 20 minutes? 45 min.? 10 min.? 50 min.? 24 min.? 15 min.?

3. What part of a bu. is 3 pk.? 6 qt.? 2 pk.?

4. What part of a foot is 3 in.? 9 in.? 7 in.? 11 in.?

5. What part of a £ is 4 s.? 9 s.? 12 s.? 15 s.?

6. What part of a gallon is 3 qt.? 2 pt.? 1 pt.?

7. What fraction of 6 lb. (Troy) is 18 oz.? 24 oz.? 9 oz.?

8. What fraction of an acre is 75 sq. rd.? 80 sq. rd.?

9. What part of 3 yd. is 8 ft.? 6 ft.? 5 ft.?

10. What part of a bu. is 2 pk. 3 qt.? 1 pk. 7 qt.?

WRITTEN EXERCISES.

1. Reduce 37 min. 30 sec. to the fraction of an hour.

OPERATION.

37 min. 30 sec. = 2250 sec.

1 hr. = 3600 sec.

$\frac{2250}{3600} = \frac{5}{8}$ hr., *Ans.*

Or,

30 sec. = $\frac{30}{60}$ min. = $\frac{1}{2}$ min.

$$37\tfrac{1}{2} \text{ min.} = \frac{\overset{5}{\cancel{75}}}{2 \times \underset{4}{\cancel{60}}} = \frac{5}{8} \text{ hr., } \textit{Ans.}$$

SOLUTION.—37 min. 30 sec. reduced to seconds give 2250 seconds; whereas in 1 hr. there are 3600 seconds; hence, 37 min. 30 sec. equal to $\frac{2250}{3600}$ hr. = $\frac{5}{8}$ hr. Or,

Since 60 sec. = 1 min., 30 sec. = $\frac{30}{60}$ min., or $\frac{1}{2}$ min.; which added to 37 gives 37½ min. And since 60 min. = 1 hr., 37½ min. = $\frac{75}{2 \times 60}$ hr. = $\frac{5}{8}$ hr., *Ans.*

2. Reduce 6 sq. ft. 108 sq. in. to a fractional sq. yd.

OPERATION.

108 sq. in. ÷ 144 = .75 sq. ft.
6.75 sq. ft. ÷ 9 = .75 sq. yd., *Ans.*

SOLUTION.—Dividing 108 sq. in. by 144, the number of sq. in. in a sq. ft., we obtain .75 of a sq. ft.; which added to 6 sq. ft. gives 6.75. This divided by 9, the number of sq. ft. in a sq. yd., gives .75 sq. yd., *Ans.*

NOTE.—The fractional part may be obtained by either of the solutions in problem *1*, and may then be reduced to the required decimal.

Reduce :

3. 8s. 8d. to the fraction of a £. *Ans.* $\frac{13}{30}$ of a £.
4. 9 oz. 15 pwt. to the fraction of a lb.
5. 1 ft. 4 in. to the fraction of a yd. *Ans.* $\frac{4}{9}$ of a yd.
6. 13 rd. 10 ft. 8 in. to the fraction of a mile.
7. 14s. 6d. to the decimal of a £. *Ans.* .725 of a £.
8. 18 lb. 10 oz. to the decimal of a cwt.

ADDITION.

173. ***Addition of Compound Numbers*** is the process of finding the sum of two or more similar compound numbers.

1. Find the sum of 5 T. 6 cwt. 73 lb.; 8 cwt. 80 lb.; 15 T. 3 cwt. 11 oz.; and 10 T. 8 cwt. 49 lb. 10 oz.

OPERATION.

T.	cwt.	lb.	oz.
5	6	73	0
	8	80	0
15	3	0	11
10	8	49	10
31	7	3	5

SOLUTION.—Arrange the quantities so that units of the same denominations may stand in the same column. Beginning with the lowest denomination, we find that 10 oz. + 11 oz. = 21 oz. (16 oz. = 1 lb.), or 1 lb. 5 oz. Place the 5 oz. in the ounce column, and add the 1 lb. to the lb. of the next column. 1 lb. + 49 lb. + 80 lb. + 73 lb. = 203 lb., (100 lb. = 1 cwt.) or 2 cwt. 3 lb. Writing the 3 lb. in the proper column, we

add the 2 cwt. to the column of cwt.; 2 cwt. + 8 cwt. + 8 cwt. + 8 cwt. + 6 cwt. = 27 cwt. (20 cwt. = 1 T.), or 1 T. 7 cwt. Placing 7 cwt. in the cwt. column, add 1 T. to the tons of the next column. 1 T. + 10 T. + 15 T. + 5 T. = 31 T. Having no higher denomination, we put down the entire amount. The sum then is 31 T. 7 cwt. 3 lb. 5 oz.

WRITTEN EXERCISES.

2. Find the sum of 45° 55′ 43″; 16° 45′ 54″; and 103° 57′ 8″. *Ans.* 166° 38′ 45″.

3. Find the sum of 17 da. 14 hr. 35 min.; 39 da. 8 hr. 50 min.; and 53 da. 13 hr. 25 min.

4. A London clothier's receipts for 5 weeks were as follows: £44 18s. 9d.; £99 19s. 10d. 2 far.; £39 8s. 8d.; £29 10s. 9d. 2 far.; £10 4s. 3d. 1 far. What was the entire amount? *Ans.* £224 2s. 4d. 1 far.

5. Find the sum of 18 T. 16 cwt. 50 lb. 8 oz.; 20 T. 10 cwt. 40 lb. 10 oz.; 67 lb. 11 oz.; 13 T. 60 lb. 15 oz.; 60 T. 7 cwt.; 103 T. 7 cwt. 78 lb. 9 oz. *Ans.* 216 T. 2 cwt. 98 lb. 5 oz.

6. How much wine in four casks containing, respectively 8 gal. 2 qt. 1 pt.; 10 gal. 3 qt. 3 gi.; 3 gal. 1 pt. 2 gi.; and 15 gal. 3 qt. 1 pt. 3 gi.?

7. Add 20 bu. 3 pk. 5 qt. 1 pt.; 18 bu. 2 pk. 7 qt.; 27 bu. 1 pt.; 6 qt. 1 pt.; 1 bu. 1 pk. 1 qt. 1 pt.; and 3 pk. 1 pt. *Ans.* 68 bu. 3 pk. 5 qt. 1 pt.

8. Add 8° 27′ 45″; 12° 30′ 17″; 90° 10′ 30″; 25° 14′ 35″; 190° 50′ 58″; and 10′ 45″.

9. Find the sum of 14 hhd. 30 gal. 2 qt.; 50 gal. 2 qt. 1 pt.; 18 hhd. 37 gal. 3 qt.; 10 hhd. 25 gal.; 1 hhd. 36 gal.; and 16 hhd. 13 gal. 4 qt.

10. Find the sum of 8 yr. 3 mo. 16 da.; 4 yr. 11 mo. 29 da.; 10 yr. 8 mo. 10 da.; 1 yr. 1 mo. 6 da.; and 9 mo. 21 da. *Ans.* 25 yr. 10 mo. 12 da.

11. Add 8 da. 3 hr. 4 min. 50 sec.; 16 hr. 4 min. 18 sec.; 10 da. 3 hr. 20 min.; and 20 da. 6 hr. 3 min. 30 sec.
Ans. 39 da. 4 hr. 32 min. 38 sec.

12. Add 3 T. 4 cwt. 70 lb.; 3 cwt. 60 lb. 10 oz.; 3 T. 18 lb.; 14 cwt. 8 oz.; 8 T. 9 cwt. 16 lb. 14 oz.; and 18 lb. 12 oz. *Ans.* 15 T. 11 cwt. 84 lb. 12 oz.

13. A farmer sold 16 lb. 3 oz. of butter on Monday; 27 lb. 10 oz. on Tuesday; 10 lb. 9 oz. on Wednesday; 40 lb. 6 oz. on Thursday; 48 lb. 9 oz. on Friday; and 30 lb. 10 oz. on Saturday. How much butter did he sell in that week? *Ans.* 1 cwt. 73 lb. 15 oz.

14. Find the sum of 10 lb. 10 oz. 3 pwt. 8 gr.; 11 oz. 16 gr.; 3 lb. 8 oz. 16 pwt.; 9 oz. 6 pwt. 14 gr.; 10 lb. 8 oz. 10 pwt.; and 13 lb. 6 oz. 10 pwt. 18 gr.
Ans. 40 lb. 6 oz. 7 pwt. 8 gr.

15. Add 8 m. 80 rd. 5 yd. 1 ft.; 2 m. 17 rd. 3 yd. 2 ft.; and 11 m. 13 rd. 2 yd. 2 ft.

16. A pile of wood contains 50 cd. 16 cu. ft.; another 90 cd. 75 cu. ft.; a third 100 cd. 100 cu. ft.; and a fourth 75 cd. and 110 cu. ft. How much wood in the four piles?
Ans. 317 cd. 45 cu. ft.

17. How much land in 5 farms containing, respectively, 186 A. 3 sq. rd.; 100 A. 40 sq. rd.; 163 A. 130 sq. rd.; 42 A. 16 sq. rd.; and 150 A. 100 sq. rd.?
Ans. 1 sq. m. 2 A. 129 sq. rd.

18. Find the sum of 31 cu. yd. 10 cu. ft. 378 cu. in.; 444 cu. yd. 19 cu. ft. 520 cu. in.; 258 cu. yd. 16 cu. ft. 300 cu. in.; and 173 cu. yd. 25 cu. ft. 1000 cu. in.

19. A farmer raised 10 bu. 3 pk. 5 qt. of corn in one field; 30 bu. 3 pk. 6 qt. in a second field; 25 bu. 7 qt. in a third field; and 70 bu. 6 pk. in a fourth field. How much corn did he raise in the four fields?
Ans. 138 bu. 2 pk. 2 qt.

SUBTRACTION.

174. ***Subtraction of Compound Numbers*** is the process of finding the difference between two similar compound numbers.

1. What is the difference between 90° 2′ 30″ and 5′ 38″?

OPERATION.

°	′	″
90	2	30
	5	38
89	56	52

SOLUTION.—The numbers are so arranged that units of the same denomination stand in the same column. As we cannot subtract 38″ from 30″ we add to this latter number 1 unit of the next higher denomination, which gives 30″ + 60″ = 90″; then 90″ minus 38″ leaves 52″. In order that the difference may not be changed, having "borrowed" 1′, it is necessary to add 1′ to 5′; but as 6′ cannot be subtracted from 2′, add 1° or 60′ to the 2′, which gives 62′. 62′ less 6′ equals 56′; and lastly, the 1° "borrowed," is taken from 90°, leaving a remainder of 89°, which completes the work.

WRITTEN EXERCISES.

2. From £50 6s. 8d. take £45 7s. 5d.

3. From a barrel of molasses, containing 59 gal. 2 qt. 1 pt., a grocer sold 37 gal. 3 qt. 1 pt.; how much had he left? *Ans.* 21 gal. 3 qt.

4. A tree 16 yd. 2 ft. 3 in. high, was struck by lightning and broken off 6 yd. 1 ft. 10 in. from the top. What was the height of the part left standing?

5. How much remains of 50 bushels of corn, after selling 36 bu. 2 pk. 5 qt.? *Ans.* 13 bu. 1 pk. 3 qt.

6. From a barrel of wine containing 31 gal. 2 qt., there leaked out 11 gal. 3 qt.; how much remained?

7. How much remains of a farm of 175 A. 16 sq. rd. 20 sq. yd., after selling 90 A. 27 sq. rd. 9 sq. yd.?

DIFFERENCE BETWEEN DATES.

8. Find the difference in time from October 27th, 1860, to March 31st, 1883.

OPERATION.

yr.	mo.	da.
1883	3	31
1860	10	27
22	5	4

SOLUTION.—Observing that October is the 10th month, and March is the 3d, we arrange the denominations in order, and subtract, 30 days being counted to a month and 12 months to a year.

Find the difference in time:

9. From May 10th, 1876, to November 10th, 1876.
10. From April 19th, 1775, to July 4th, 1776.
11. From July 4th, 1776, to October 19th, 1781.
12. From April 14th, 1861, to April 26th, 1865.
13. From May 7th, 1873, to March 11th, 1878.
14. From June 6th, 1880, to Sept. 2d, 1882.
15. From Sept. 17th, 1879, to Feb. 8th, 1883.
16. From Jan. 11th, 1864, to May 7th, 1881.
17. From July 4th, 1880, to March 4th, 1883.
18. From Feb. 9th, 1860, to April 7th, 1883.
19. From Oct. 11th, 1863, to June 9th, 1882.

LONGITUDE AND LATITUDE.

175. ***The Longitude*** of a place is its distance east or west of a given meridian.

176. ***The Latitude*** of a place is its distance north or south of the equator.

177. Longitude and Latitude are reckoned in degrees, minutes, and seconds. The highest degree of longitude is 180 degrees; and the highest degree of latitude is 90 degrees.

TABLE

Of the Longitude (meridian of Greenwich) of a few important places.

1. Albany, 73° 44′ 50″ W.
2. Boston, 71° 3′ 30″ W.
3. Berlin, 13° 23′ 45″ E.
4. Cincinnati, 84° 29′ 31″ W.
5. Chicago, 87° 37′ 45″ W.
6. Constantinople, 28° 59′ 0″ E.
7. Jerusalem, 35° 32′ 0″ E.
8. London, 0° 51′ 38″ W.
9. Mexico, 99° 5′ 0″ W.
10. Montreal, 73° 25′ 0″ W.
11. New York, 74° 0′ 8″ W.
12. New Orleans, 90° 2′ 30″ W.
13. Paris, 2° 20′ 22″ E.
14. Philadelphia, 75° 10′ 0″ W.
15. Pekin, 116° 23′ 45″ E.
16. Rome, 12° 27′ 14″ E.
17. S. Francisco, 122° 26′ 45″ W.
18. St. Louis, 90° 15′ 15″ W.
19. Washington, 77° 0′ 15″ W.
20. St. Petersburg, 30° 19′ 0″ E.

TABLE

Showing the Latitude of a few places.

1. Buenos Ayres, 34° 3′ S.
2. Cape of Good Hope, 34° 22′ S.
3. Cape Horn, 55° 58′ 40″ S.
4. Charleston, S.C., 32° 46′ 33″ N.
5. Copenhagen, 55° 40′ N.
6. Cape Town, 34° 56′ S.
7. Dublin, 53° 21 N.
8. Mozambique, 15° 2′ S.
9. New York, 40° 42′ 43″ N.
10. New Orleans, 29° 58′ N.
11. Paris, 48° 50′ N.
12. Philadelphia, 39° 57′ N.
13. Rome, 41° 54′ N.
14. St. Louis, 38° 27′ 28″ N.

1. Find the difference in longitude between Albany and Boston.

OPERATION.

Albany, 73° 44′ 50″ W.
Boston, 71° 3′ 30″ W.
Ans. 2° 41′ 20″

SOLUTION.—Arranging the denominations in order, we subtract, both places being on the same side of the meridian.

NOTES.—1. If the places be on different sides of the meridian, take their sum.

2. To obtain the difference of Latitude subtract, if both places are in the same direction from the equator; if in different directions, add.

3. When the difference of either latitude or longitude is found by adding, and the sum be greater than 180°, subtract it from 360°.

WRITTEN EXERCISES.

Find the difference in longitude between:

2. Boston and Cincinnati; Chicago and Mexico.
Ans. 13° 26′ 1″; 11° 27′ 15″.

3. Montreal and New York; Philadelphia and Washington. *Ans.* 35′ 3″; 1° 50′ 15″.

4. Constantinople and Jerusalem; Paris and Pekin.
Ans. 6° 33′; 114° 3′ 23″.

5. Berlin and London; Paris and New York.
Ans. 14° 15′ 23″; 76° 20′ 25″.

6. Rome and St. Louis; San Francisco and St. Petersburg. *Ans.* 102° 42′ 29″; 152° 45′ 45″.

Find the difference in latitude between:

7. Charleston, S. C., and Copenhagen; New Orleans and New York. *Ans.* 22° 53′ 27″; 10° 44′ 43″.

8. St. Louis and Mozambique; Dublin and Paris.
Ans. 53° 29′ 28″; 4° 31′.

9. Buenos Ayres and Cape Town; Cape Horn and Rome. *Ans.* 53′; 97° 52′ 40″.

10. Philadelphia and Charleston, S. C.; Philadelphia and Cape of Good Hope. *Ans.* 7° 10′ 27″; 74° 19′.

MULTIPLICATION.

178. ***Multiplication of Compound Numbers*** is the process of finding the product when the multiplicand
mpound number.

Vhat are the contents of 5 cisterns, each containing
2 qt. 1 pt.?

OPERATION.

gal.	qt.	pt.
50	2	1
		5
253	0	1

SOLUTION.—Commence at the right. 5 times 1 pt. gives 5 pt. (2 pt. = 1 qt.), or 2 qt. and 1 pt. Write 1 pt. and carry 2 qt.; 5 times 2 qt. gives 10 qt. and 2 qt. carried make 12 qt. (4 qt. = 1 gal.), or 3 gal. Having no remainder, place 0 in the place of qt.; 5 times 50 gal. = 250 gal., and 3 gal. carried = 253 gal.; which completes the product.

EXERCISES.

2. Multiply £137 18s. 4½d. by 25.
Ans. £3447 19s. 4½d.

3. Multiply 8 da. 6 hr. 7 min. 48 sec. by 84.
Ans. 1 yr. 328 da. 10 hr. 55 min. 12 sec.

4. Multiply 8 yd. 2 ft. 10 in. by 18. *Ans.* 161 yd.

5. Multiply £12 13s. 10d. by 180. *Ans.* £2284 10s.

6. Multiply 3 lb. 6 oz. 7 dr. 4 scr. by 16.
Ans. 57 lb. 4 oz. 5 dr. 1 scr.

7. Multiply 16° 9′ 8″ by 20. *Ans.* 323° 2′ 40″.

8. Multiply 80° 30′ 30″ by 4. *Ans.* 322° 2′.

9. What are the contents of 6 casks of wine, each containing 67 gal. 3 qt. 1 pt. 2 gi.?
Ans. 407 gal. 2 qt. 1 pt.

10. How much coal in 3 cars, each containing 6 T. 8 cwt. 75 lb.? *Ans.* 19 T. 6 cwt. 25 lb.

11. If a spoon contain 3 oz. 6 pwt. 20 gr. of silver, what will 12 spoons weigh? *Ans.* 3 lb. 4 oz. 2 pwt.

12. If a man travel 35 m. 101 rd. 1½ ft., consecutively, for 6 days, how far has he travelled?
Ans. 211 m. 286 rd. 9 ft.

13. What is the total weight of 32 sheep, having an average weight of 69 lb. 11 oz.? *Ans.* 22 cwt. 30 lb.

14. How far will a railroad train go in 16 hr. at the rate of 35 m. 103 rd. 1½ yd. per hr.?
Ans. 565 m. 52 rd. 2 yd.

15. How much wine in 12 casks, each containing 37 ga 3 qt. 1 pt.? *Ans.* 454 gal. 2 qt.

16. If a steam-boat run a mile in 5 min. 26 sec., ho long will it take to make a trip of 98 m.?
Ans. 8 hr. 52 min. 28 sec.

17. If a meteor move 6° 27′ 50″ in a second, how far will it move in 25 seconds? *Ans.* 161° 35′ 50″.

18. How much hay in 13 loads, of 1 T. 3 cwt. 48 lb. each? *Ans.* 15 T. 5 cwt. 24 lb.

19. In 12 bins, with a capacity of 200 bu. 3 pk. 6 qt. each, how much corn? *Ans.* 2411 bu. 1 pk.

20. How much land in 37 farms of 16 A. 6 sq. rd. 20 sq. yd. each? *Ans.* 593 A. 86 sq. rd. 14 sq. yd.

DIVISION.

179. ***Division of Compound Numbers*** is the process of finding the quotient when the dividend is a compound number.

1. Divide £113 1s. 10d. 2 far. by 21.

OPERATION.

```
       £    s.   d.  far.  £   s.  d. far.
  21 ) 113  1   10   2 ( 5,  7,  8,  2.
       105
       ---
         8             21 ) 178 ( 8
        20                  168
       ---                  ---
  21 ) 161 ( 7               10
       147                    4
       ---                  ---
        14             21 ) 42 ( 2
        12                  42
       ---                  ---
       178
```

SOLUTION.—Dividing £113 by 21 we obtain £5, and a remainder of £8, which, reduced to shillings, and added to 1s. of the dividend, gives 161s. This divided by 21 gives 7s. and a remainder of 14s. Reducing this last to pence and adding it to 10d. gives 178d.; which divided by 21 gives 8d. and a remainder of 10d. 10d. = 40 far. And 40 far. added to 2 far. gives 42 far. This divided by 21 gives 2 far. and no remainder.

WRITTEN EXERCISES.

2. Divide 309° 27′ 30″ by 25. *Ans.* 12° 22′ 42″.

3. Divide 71 lb. 0 oz. 8 pwt. by 16.
Ans. 4 lb. 5 oz. 5 pwt. 12 gr.

4. Divide £650 1s. 4d. by 196. *Ans.* £3 6s. 4d.

5. Divide 524 gal. 2 qt. 1 pt. 2 gi. by 365.
Ans. 1 gal. 1 qt. 1 pt. 2 gi.

6. Divide 32 bu. 3 pk. 6 qt. by 24.
Ans. 1 bu. 1 pk. 3 qt. 1$\frac{5}{6}$ pt.

7. Divide 19 cwt. 88 lb. 12 oz. by 10.
Ans. 1 cwt. 98 lb. 14 oz.

8. Divide £183 10s. 1$\frac{1}{2}$d. by 18.
Ans. £10 3s. 10$\frac{3}{4}$d.

9. 11 lb. 1 oz. 4 pwt. of silver were used in making 18 goblets; how much silver was used for each?
Ans. 7 oz. 8 pwt.

10. How many packages, each containing 5 oz., can be filled from 95 lb. of cinnamon? *Ans.* 304 packages.

NOTE.—When both dividend and divisor are compound numbers reduce them to the same denomination and divide as in simple numbers.

11. A vessel sailed 258 m. 196 rd. 9 ft. in 21 hours; how far was that per hour? *Ans.* 12 m. 100 rd. 13 ft.

12. Divide 1275 A. 96 sq. rd. 22 sq. yd. 8 sq. ft. 32 sq. in. equally among 32 persons.
Ans. 39 A. 138 sq. rd. 6 sq. ft. 64 sq. in.

13. If a town 6 miles square be divided into 165 farms, how many acres will each farm contain?
Ans. 139 A. 101 sq. rd. 24 sq. yd. 6 sq. ft. 108 sq. in.

14. How many bins, each containing 65 bu. 1 pk. 7 qt. 1 pt. will be required to contain 785 bu. 3 pk. 2 qt.?
Ans. 12 bins.

15. How far will a meteor move in a second, if it move 242° 26′ 15″ in 45 seconds? How far will it move in 25 seconds?

16. How many loads of wood in 69 cd. 12 cu. ft., each load containing 2 cd. 12 cu. ft.? *Ans.* 33 loads.

17. If a man travel 3 m. 108 rd. in an hour, how long will it take him to travel 36 m. 228 rd.?

Ans. 11 hours.

18. Divide 114 cwt. 30 lb. 8 oz. by 8.

Ans. 14 cwt. 28 lb. 13 oz.

19. The aggregate weight of 67 hhd. of sugar is 28 T. 18 cwt. 21 lb. What is the average weight of a hhd.?

Ans. 8 cwt. 63 lb.

20. From 10 lb. 8 oz. 6 dr. 1 scr. of camphor, how many packages can be put up, each containing 3 dr. 2 scr.?

Ans. 281 packages.

LONGITUDE AND TIME.

180. As there are 360° in a circumference, and the earth makes a revolution on its axis once every 24 hours, it follows that in every hour the earth proceeds 15°; in a minute $\frac{1}{60}$ of 15°, or 15′; and in a second $\frac{1}{60}$ of 15′, or 15″. Hence, also, the

TABLE.

1 hour of time = 15° of longitude.
1 minute of time = 15′ of longitude.
1 second of time = 15″ of longitude.

NOTE.—The *Standard Time* of the United States and Canada is divided into the Eastern, Central, Mountain, and Pacific time. It is reckoned from the four meridians, 75°, 90°, 105°, and 120° west of Greenwich. As 15° of longitude equal 1 hour of time, Eastern time is 5 hours slower than London time.

CASE I.

181. To find the difference in time between two places when the difference in longitude is given.

1. The difference of longitude between two places is 70°; what is the difference in time?

OPERATION.

$$15\,)\,\underline{70° \quad 0'}$$
$$4 \text{ hr. } 40 \text{ min.}$$

SOLUTION.—Since 15° of longitude are equal to one hour of time, 70° of longitude are equal to 4 hours, and a remainder of 10°, or 600′. 600′ are equal to 40 minutes. since 15′ of longitude are equal to 1 minute. Hence, the difference in time is 4 hr. 40 min.

WRITTEN EXERCISES.

Find the difference in time between two places whose difference of longitude is:

2. 18° 40′. *Ans.* 1 hr. 14 min. 40 sec.
3. 150°. *Ans.* 10 hr.
4. 28° 18′ 15″. *Ans.* 1 hr. 53 min. 13 sec.
5. 30° 40′ 30″. *Ans.* 2 hr. 2 min. 42 sec.
6. 100° 20′. *Ans.* 6 hr. 41 min. 20 sec.
7. 80° 45′. *Ans.* 5 hr. 23 min.

Find the difference in time between:

8. Albany and Montreal; Constantinople and Jerusalem. *Ans.* 1 min. 19 sec.; and 26 min. 12 sec.

9. Mexico and London; New Orleans and San Francisco. *Ans.* 6 hr. 32 min. 53 sec.; 2 hr. 9 min. 37 sec.

10. St. Louis and Berlin; Philadelphia and Pekin.
Ans. 6 hr. 54 min, 36 sec.; 12 hr. 46 min. 15 sec.

11. Washington and Paris; St. Petersburg and Paris.
Ans. 5 hr. 17 min. 22 sec.; 1 hr. 51 min.

12. New York and Cincinnati; Chicago and Boston.

Ans. 41 min. 57 sec.; and 1 hr. 6 min. 17 sec.

13. What change must be made in our watches, if we travel from Berlin to Paris?

Ans. Set back 44 min. 13 sec.

NOTE.—To find the time of a place *east*, add the difference of time between the two places to the given time. But to find the time of a place *west*, subtract the difference of time between the two places from the given time.

14. What time is it in Boston when it is 4 o'clock P.M. in Mexico? *Ans.* 52 min. 6 sec. past 5 P.M.

15. When it is noon at Greenwich, what time is it at the first two places given in the table?

Ans. To first, 5 min. ⅔ sec. past 7 A.M.

16. When it is half past seven A.M. in New York, what time is it in Philadelphia? New Orleans? Cincinnati? Paris? London?

Ans. To last, 22 min. 33⅔ sec. past 12 noon.

17. What change would a person have to make in his watch, in going from Washington to London?

Ans. Set ahead, 5 hr. 4 min. 34 sec.

CASE II.

182. To find the difference of longitude when the time is given.

1. The difference in time between two places is 4 hr. 10 min.; what is the difference of longitude?

OPERATION.

```
4 hr.  10 min.
       15
─────────────
62°    30′
```

SOLUTION.—Since an hour of time equals 15° of longitude, and a minute of time equals 15′ of longitude, 15 times the number of hours and minutes will equal the difference of longitude in degrees and minutes. Hence, 15 times 4 hr. 10 min. equals 62° 30′.

WRITTEN EXERCISES.

Find the difference in longitude of two places whose difference of time is:

2. 3 hr. 20 min. *Ans.* 50°.

3. 36 min. *Ans.* 9°.

4. 4 hr. 22 min. *Ans.* 65° 30′.

5. 8 hr. 16 min. 24 sec. *Ans.* 124° 6′.

6. 6 hr. 6 min. 30 sec. *Ans.* 91° 37′ 30″.

7. It is 6 hr. 52 min. 40 sec. A.M. in Harrisburg, Penn., when it is noon at Greenwich; what is its longitude? *Ans.* 76° 50′.

8. When it is 8 o'clock A.M. in New Orleans, it is 9 hr. 5 min. 11 sec. in Albany; what is the difference of longitude? *Ans.* 16° 17′ 45″.

TEMPERATURE.

183. A ***Thermometer*** is an instrument for measuring variations of heat and temperature.

There are three methods of registering heat and temperature by means of thermometers.

Fahrenheit's, used in the United States, England, and Holland; Réaumer's, used in Russia, Spain, and Turkey; and Celsius's, called the Centigrade, used in France, Germany, Sweden, and some other countries of Europe.

In Fahrenheit's (F.) the freezing point is marked 32°, and the boiling point 212°.

In Réaumer's (R.) the freezing point is zero, and the boiling point 80°.

The Centigrade (C.) has the freezing point zero, and the boiling point 100°.

184. To reduce from one scale to another.

RULE I.—*To express degrees Fahrenheit in Réaumer, deduct 32 and take ⁴⁄₉ of the remainder; conversely, to change Réaumer to Fahrenheit, multiply by ⁹⁄₄ and add 32 to the product.*

Formulas: $R. = \frac{4}{9}(F. - 32°)$. $F. = \frac{9}{4}R. + 32°$.

ILLUSTRATIONS.

1. Express 59° F. in Réaumer's scale.

2. Express 24° R. in Fahrenheit's scale.

OPERATION *1.*

$59° - 32° = 27°$ F.

$\frac{4}{\cancel{9}} \times \overset{3}{\cancel{27}}°$ F. = 12° R., *Ans.*

OPERATION *2.*

$\frac{9}{\cancel{4}} \times \overset{6}{\cancel{24}}°$ R. = 54°.

$54° + 32° = 86°$ F., *Ans.*

WRITTEN EXERCISES.

3. Express in Réaumer, 68° F.; 95° F.; 77° F.; 140° F.
Ans. 16° R.; 28° R.; 20° R.; 48° R.

4. Express in Fahrenheit, 32° R.; 76° R.; 8° R.; 50° R.
Ans. 104° F.; 203° F.; 50° F.; 144½° F.

5. Express 47° F. in R.; 59° R. in F.; 67° R. in F.; 14° F. in R.; 10° R. in F.
Ans. 6⅔° R.; 164¾° F.; 182¾° F.; —8° R.; 54½° F.

6. Express 5° F. in R.; —4° R. in F.; 252° R. in F.
Ans. —12° R.; 23° F.; 599° F.

RULE II.—*To express degrees Fahrenheit in Centigrade, subtract 32 and take ⁵⁄₉ of the remainder; conversely, to change Centigrade to Fahrenheit, multiply by ⁹⁄₅ and add 32 to the product.*

Formulas: $C. = \frac{5}{9}(F. - 32°)$. $F. = \frac{9}{5}C. + 32°$.

ILLUSTRATIONS.

1. Express in Centigrade 77° F.

2. Express in Fahrenheit 75° C.

OPERATON *1.*

$77° - 32° = 45°$ F.

$\frac{5}{\cancel{9}} \times \overset{5}{\cancel{45}}° = 25°$ C., *Ans.*

OPERATION *2.*

$\overset{15}{\cancel{75}}° \times \frac{9}{\cancel{5}} = 135°.$

$135° + 32° = 167°$ F., *Ans.*

WRITTEN EXERCISES.

3. Express in Centigrade, 95° F.; 131° F.; 41° F.; 203° F. *Ans.* 35° C.; 55° C.; 5° C.; 95° C.

4. Express in Fahrenheit, 15° C.; 90° C.; 130° C.; 30° C. *Ans.* 59° F.; 194° F.; 266° F.; 86° F.

5. Express in C. 14° F. In C. 71° F. In F. 18° C. In F. 91° C. *Ans.* −10° C.; 21.67° C.; 64.4° F.; 195.8° F.

Rule III.—*To express degrees Réaumer in Centigrade, multiply by $\frac{5}{4}$; and, conversely, to change Centigrade to Réaumer, multiply by $\frac{4}{5}$.*

Formulas: $C. = \frac{5}{4} R.$ $R. = \frac{4}{5} C.$

ILLUSTRATIONS.

1. Express in Centigrade 60° R.

2. Express in Réaumer 50° C.

OPERATION *1.*

$\frac{5}{\cancel{4}} \times \overset{15}{\cancel{60}}° = 75°$ C., *Ans.*

OPERATION *2.*

$\overset{10}{\cancel{50}}°$ C. $\times \frac{4}{\cancel{5}} = 40°$ R., *Ans.*

WRITTEN EXERCISES.

3. Express in Centigrade 36° R.; 80° R.; 120° R.; 40° R. *Ans.* 45° C.; 100° C.; 150° C.; 50° C.

4. Express in Réaumer 80° C.; 160° C.; 320° C.; 60° C. *Ans.* 64° R.; 128° R.; 256° R.; 48° R.

PRACTICAL MEASUREMENTS.

185. These ***Practical Measurements*** include measures of surface, volume, capacity, lumber, wood, stone and brick work, shingles, laths, plastering, painting, and carpeting.

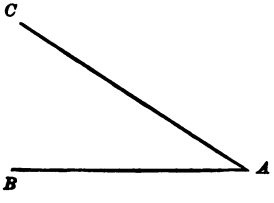
186. A ***Line*** is that which has only one dimension, *length.*

187. A ***Surface*** is that which has two dimensions, *length* and *breadth.* When it does not change its direction, it is called *plane.*

188. A ***Polygon*** is a portion of a plane surface bounded by straight lines.

189. The ***Perimeter*** of a polygon is the sum of all its bounding lines, or sides.

190. The ***Area*** of a polygon is the surface included within its sides.

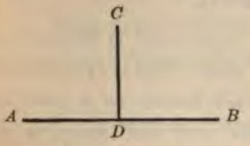


191. An ***Angle*** is the opening or inclination of two straight lines meeting each other in a common point, called the *vertex;* as *BAC.*

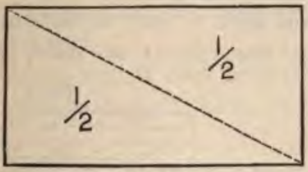
Angles are either *right*, *acute*, or *obtuse.* A *Right Angle* is an angle of 90 degrees. An *Acute Angle* is less than a right angle; and an *Obtuse Angle* is greater than a right angle.

192. A ***Perpendicular Line*** is a line drawn to meet another in such a manner as to incline no more to one side than to the other. The two adjacent angles thus formed are equal, and are called *Right Angles.*

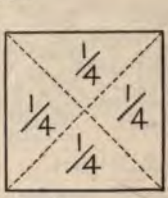
In the figure, the line *CD* is perpendicular to *AB*, and the angles *ADC* and *BDC* are equal, and right angles.



193. A ***Rectangle*** is a polygon bounded by four straight lines and having four right angles.



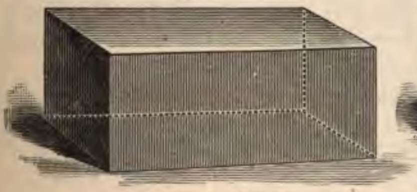
Rectangle.



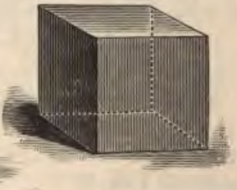
Square.

194. A ***Square*** is a rectangle which has all its sides equal.

195. A ***Solid,*** or volume, is that which has length, breadth, and thickness, or height. The plane surfaces bounding a solid are called *Faces.*



Rectangular Solid.



Cube.

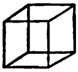
196. A ***Cube*** is a solid bounded by six equal squares.

197. A ***Rectangular Solid,*** or parallelopipedon, is a volume bounded by rectangular faces; as, a box, a room, a block of marble, a load of wood.

198. The ***Measurement*** of a line, of a surface, or of a solid consists in finding how many times it contains another magnitude of the same kind taken as a *unit of measure.* Thus, to say that a line is 6 feet long, is the same as to say that it contains the *unit of measure,* 1 foot, 6 times.

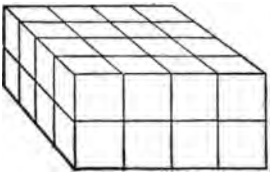
199. The ***Measuring Unit*** of a solid is a small cube, or solid, whose side is one inch, one foot, or any other convenient length. The number of times such a unit is contained in any solid, gives the *volume* or *solid contents* of that solid.

Fig. 1.



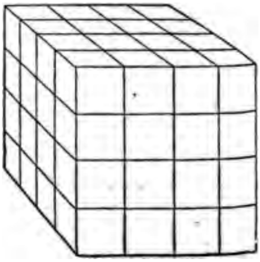
1 cu. ft.

Fig. 2.



32 cu. ft.

Fig. 3.



64 cu. ft.

Thus, the *measuring unit* of the solid represented in the margin, Fig. 8, is a small cube, Fig. 1, which is 1 ft. long, **1 ft. wide,** and 1 ft. thick. Its solidity is equal to the product of **its three dimensions,** or 1 cu. ft.

The given solid, Fig. 3, may be divided into 2 equal parts, each of which is 4 ft. long, 4 ft. wide, and 2 ft. thick, as shown in Fig. 2. These parts may be subdivided into 32 smaller solids, whose dimensions are the same as those of the unit of measure. Hence, the measure of the given solid equals 4 ft. × 4 ft. × 4 ft. = 64 cu. ft.

WRITTEN EXERCISES.

SURFACE.

1. How many trees planted 10 ft. apart, will be required to go round a quarter-section of land?

Ans. 1056 trees.

NOTE.—A Section of land is one mile square.

2. What is the cost of a piece of oil-cloth 30 ft. long by 18 ft. 6 in. wide, at 60c. a square yard? *Ans.* $37.

3. A 20-acre field is 80 rods long. How many rods of fence are needed to enclose the field?

Ans. 240 rods.

4. Find the cost of 121000 feet of plank, at $28.25 per thousand feet. *Ans.* $3418.25.

5. What will it cost to cover a table 9 ft. long, 2¼ ft. wide, with baize ¾ yd. wide, @ 75c. per yd.?

Ans. $2.25.

6. What will it cost to kalsomine a ceiling 48 ft. by 25 ft. 4 in., at 8c. a square yard? *Ans.* $10.81.

7. What will 5400 sq. rods of land cost, at $2.50 per square chain? *Ans.* $843.75

8. How many garden plots, each containing 12 sq. yd., can be made out of a two-acre lot, allowing ⅛ of the lot for walks? *Ans.* 705⅚ plots.

9. What is the value of a township of land, at $140 a half-section? *Ans.* $10080.

NOTE.—A Township is six miles square.

10. How many quarter-sections are there in a piece of land 32 miles long by 5 miles wide?

Ans. 640 quarter-sections.

11. How many square rods in three quarter-sections of land? *Ans.* 76800 sq. rd.

12. How many sq. yd. in the walls and ceiling of a room 26 ft. long, 18 ft. wide, and 13 ft. high?

13. How much will it cost to pave a sidewalk 145 ft. long and 8 ft. wide, at 75c. a sq. yd.?

14. How many posts, placed 8 ft. apart, are required for a fence round a garden 236 yards by 124 yards?

15. In No. 14, find the cost of the posts, at 15c. apiece.

16. How many 2½-inch pickets, placed 2 in. apart, will be required to fence a field 60 rods wide and 20 rods deep?
Ans. 7040 pickets.

17. What will it cost to enclose a lot, 130 rods by 50 rods, with 6 6-inch boards, at $8½ per M.?
Ans. $151.47.

18. If the perimeter of a rectangular field be 220 yd., and the difference between its length and breadth 50 yd., what is the area? *Ans.* 2400 sq. yd.

19. How many scantlings, 15 ft. long, would be required for the two stringers of a fence around the field in No. 18? *Ans.* 88 scantlings.

20. How much lumber will be required to fence the field in No. 18, with a 12-inch base-board and 4 6-inch boards? How many posts placed 12 ft. apart? *Ans.* 55 posts.

21. Find the total cost of the fence in No. 18, supposing the lumber to be $10¾ per M., the posts 12c. each, and the labor $144. *Ans.* $171.89.

22. If an acre is cut from a field 30 rods long and 10 rods wide, what is the length of the part remaining?
Ans. 14 rods.

23. How many sq. yd. of block pavement in a street ¾ of a mile long and 1¼ chains wide?

24. What will it cost to slate a roof 50 ft. long by 20 ft. wide, at $4.25 per 100 sq. ft.? *Ans.* $42.50.

VOLUME AND CAPACITY.

25. How many cu. yd. of earth will be removed by digging a cellar 45 ft. long, 28 ft. wide, and 14 ft. deep?

Ans. 653⅓ cu. yd.

26. What will be the cost of digging the cellar in No. 25, at 80c. per cu. yd., and 60c. a load for hauling?

Ans. $914.67.

NOTE.—A load of earth is equal to a cu. yd.

27. How many cu. ft. of air in a room 40 ft. 6 in. long, 28 ft. 4 in. wide, and 16 ft. 8 in. high?

Ans. 19125 cu. ft.

28. Find the contents of a bin of grain, which is 25 ft. long, 14 ft. wide, 3 ft. deep, and ⅗ full.

29. How many cu. yd. of air for each of 32 students in a room 52 ft. long, 36 ft. wide, and 12 ft. high?

30. How many building bricks can be piled in a shed 125 ft. long, 70 ft. wide, and 48 ft. high?

NOTE.—Bricks used in building are 8½ in. by 4 in. by 2½ in.

31. How many loads of gravel, spread 10 in. deep, will be required for a road 2¼ miles long, and 12 ft. wide?

Ans. 4400 loads.

32. How many pounds of water in a tank 7½ ft. long, 5 ft. wide, and 6½ ft. deep, reckoning 62½ pounds to the cubic foot? *Ans.* 15234⅜ lb.

33. If a bin is 40 ft. long, 10 ft. wide, and 8 ft. deep, how many bushels of shelled corn will it contain?

NOTE.—A bushel of grain contains about 1¼ cu. ft. In heaped measure it contains about 1⅓ cu. ft.

34. What must be the capacity of a bin that will hold 5400 bushels of potatoes? *Ans.* 6750 cu. ft.

35. How many bushels of unshelled corn in a wagon box 8 ft. long, 3½ ft. wide, and 1¼ ft. deep?

NOTE.—Two bushels of unshelled corn make one bushel of shelled corn.

36. How many bushels of turnips, heaped measure, can be stowed in a cellar 22½ ft. long, 16 ft. wide, and 7¾ ft. deep? *Ans.* 1860 bu.

37. What must be the height of a school-room that is 20 ft. long by 16 ft. wide, to allow 140 cu. ft. of air to each of 32 pupils? *Ans.* 14 ft.

38. A tank is 2¾ ft. long, 1⅝ ft. wide, and 21 inches deep; how many gallons of water will it contain?

39. At 62½c. a cu. yard, what will it cost to build a wall 84 ft. long, 15 ft. high, and 3 ft. 6 in. thick?

LUMBER AND WOOD.

40. What is the cost of a pile of wood, 24 ft. long, 4 ft. 6 in. wide, and 3 ft. 4 in. high, at $3 per cord?

41. How many cords of wood can be piled in a shed 64 ft. long, 28 ft. wide, and 20 ft. high? *Ans.* 280 cords.

42. Find the cost of a pile of wood 244 ft. 5 in. long, 24 ft. wide, and 5 ft. 4 in. high, at $4.25 a cord, and $1.00 a load for hauling. *Ans.* $1147.40.

NOTE.—Reckon 2¼ cords to a load.

43. If a cord of 2-foot wood is 5 ft. high, what is its length *Ans.* 12⅘ ft.

44. What is the value of a pile of tan-bark 60 ft. long, 24 ft. wide, and 12 ft. high, at $2.00 a cord?

45. At $16 per thousand, what will it cost to cover a bridge 220 yd. long, and 22 ft. wide, with plank 1¾ inches thick? *Ans.* $406.56.

46. How many board-feet in a plank 15 ft. long, 16 in. wide, and 1½ in. thick? How many cu. ft.?

NOTES.—1. A *board-foot* is reckoned 1 foot square and 1 inch thick, and is equal to $\frac{1}{12}$ of a cubic foot.

2. Even when the lumber is less than an inch in thickness, it is considered inch lumber.

3. In measuring the width, when a fraction is less than a half inch, it is rejected; but when it is greater than a half, it is counted a half inch.

47. How many feet of lumber in:

24 scantlings 12 ft. long, 6 in. wide, and 3½ in. thick?
10 beams 16 ft. long, and 14 in. square?
30 joists 20 ft. long, 10 in. wide, and 1¾ in. thick?
60 pieces siding 8 ft. long, 5¼ in. wide, and 1 in. thick?

48. At $15.25 per thousand, find the total cost of:

7 boards 10 ft. long, 16 in. wide, and 2½ in. thick.
3 boards 9 ft. long, 14¾ in. wide, and 3¼ in. thick.
5 boards 16 ft. long, 13 in. wide, and ⅞ in. thick.

Ans. $6.50.

49. Find the cost of the ⅞-inch flooring required for a room 36 ft. long and 20 ft. wide, at $16½ per thousand.

50. A garden is 15 rods long, and 12 rods wide; what will be the cost of a 2½-in. plank sidewalk, 3 ft. wide, around the outside of it, at $18 per thousand?

51. A bridge is 300 yd. long, and 40 ft. wide; how many feet of 2-inch plank will be required for its two foot-paths, each of which is 8 ft. wide?

52. At $12 per thousand, find the cost of covering the whole bridge in No. 51, supposing the plank on the roadway to be 4½ in. thick. *Ans.* $1512.

53. How many 4 × 5 rafters, 30 ft. long, and placed 21 in. apart, are required for a double roof 70 ft. long?

Ans. 82 rafters.

54. Find the cost of the rafters required for 3 roofs, having the same dimensions as the one in No. 53, at $14.50 per M. *Ans.* $178.35.

55. Find the cost of a close board fence, 75 rods long, 10 ft. high, and 1½ in. thick, at $22.50 per M.

STONE AND BRICK WORK.

56. How many perches of stone are there in a pile 325 ft. long, 50 ft. wide, and 16 ft. high?

NOTE.—A perch of stone = 16½ ft. × 1½ ft. × 1 ft. = 24¾ cu. ft., and a part of a perch is counted a whole one.

57. Find the cost of the stone required for a wall 25 ft. long, 12 ft. high, and 3 ft. thick, at $1.20 per perch.

58. A cellar is 32 ft. long, 20 ft. wide, and 15 ft. deep; what will be the cost of the stone required for its 20-in. walls, at $1.30 per perch? *Ans.* $145.60.

NOTE.—It is customary to measure around the outside of the wall.

59. How many cords of stone in the pile in No. 56 (100 cu. ft. = 1 cord of stone)? *Ans.* 2600 cd.

60. If a cord of stone will make 100 cu. ft. of wall, how many cords will be required for the walls in No. 58?

61. A stone house is 60 ft. front, 125 ft. deep, and 100 ft. high; the cellar walls are 15 ft. high, and 3 ft. thick. What did they cost, at 85c. a cu. yd.?

62. Find the cost of the remainder of the stone work in No. 61, at 75c. a cu. yd., supposing one half of it to be 2½ ft. thick, and the other half 1½ ft. thick.

63. Stone-masons reckon a cu. yd. of sand and 3 bu. of lime to a cord of stone. How much of each will be required for the cellar walls in No. 61?

64. Find the number of bricks required for a wall 80 ft. long, 10 ft. high, and 1½ ft. thick.

NOTES.—1. The dimensions of a brick are about $8\frac{1}{4}$ in. by 4 in. by $2\frac{1}{4}$ in.

2. To a cu. ft. we commonly reckon 27 bricks laid dry, or 20 bricks laid in mortar.

65. At $10.50 per thousand, what is the value of a pile of bricks $70\frac{5}{8}$ ft. long, 48 ft. wide, and 15 ft. high? *Ans.* $14458.50.

66. How many bricks, laid 8 in. by 4 in., will pave a walk 90 ft. long, and 15 ft. wide, with no allowance for edges or waste? *Ans.* 6075 bricks.

67. How many bricks are required for a wall 32 ft. long, 16 ft. high, and half a brick thick?

NOTE.—In *half-brick* walls, each brick has an external surface of 9 in. by 3 in.; in *single-brick* walls, $4\frac{1}{2}$ in. by 3 in., or 14 in. of surface; in *a-brick-and-a-half* walls, 10 in. of surface; and in *double-brick* walls, 7 in. of surface.

68. At $8 per thousand, find the cost of the bricks required for the double-brick walls of a basement, 75 ft. long, 40 ft. wide, and 12 ft. high. *Ans.* $454.22.

69. How much less would the bricks in No. 68 cost, if the walls were a brick and a half thick?

70. How many bricks will be required for a single brick wall 60 ft. long and 84 ft. high?

71. The front wall of a house is 125 ft. long, 75 ft. high, and 2 bricks thick. Find the cost of the bricks required, at $9 per M., allowing for 2 doors and 24 windows, each 8 ft. by 4 ft. *Ans.* $1581.674.

72. Find the number of bricks required for one course of a chimney with a flue one foot square.

Ans. 8 bricks.

NOTE.—The walls of an ordinary chimney are 4 inches thick, the bricks being laid on their broadest face.

73. A chimney is 50 ft. high, and its flue $7\frac{1}{2}$ in. by 4 in.; how many bricks were used in its construction, reckoning a mortared brick 3 in. thick?

74. Find the cost of the bricks for a chimney 30 ft. high, and having a 10-inch flue, at $12 per M.

SHINGLES AND LATHS.

75. How many bunches of shingles are required for a double roof 30 ft. long, with 16-ft. rafters?

NOTE.—1000 shingles make 4 bunches, and cover 100 sq. ft. of surface. They are 4 in. wide, are generally laid 4 in. to the weather, and are sold only in whole bunches.

76. Find the number of shingles required for a double roof 25 ft. long, with 20-ft. rafters.

77. Find the cost of the shingles in No. 76, at $4.25 per thousand. *Ans.* $42.50.

78. A roof is $87\frac{1}{2}$ ft. long, and 30 ft. wide; what will it cost to shingle it, at $4 per M.?

79. How many bunches of shingles are required for the roof in No. 78? *Ans.* 105 bunches.

80. In No. 78, how much, more or less, would it cost to use metallic shingles, at $5.25 per thousand, if each shingle covers 20 sq. in. of surface? *Ans.* $5.77 less.

81. A room is 16 ft. long, 14 ft. wide, and 12 ft. high; how many bunches of laths are required for its walls and ceiling? *Ans.* 35 bunches.

NOTE.—Laths are 4 ft. long, $1\frac{1}{2}$ in. wide, and are laid $\frac{3}{8}$ of an inch apart. A bunch contains 50, and will cover three sq. yd. of surface.

82. How many bunches of laths are required for a room 75 ft. long, 56 ft. wide, and 16 ft. high, allowing for 3 doors and 12 windows, each 14 ft. by 6 ft.?

NOTE.—Deduct only one-half of the surface of the openings.

83. Find the cost of the laths in No. 82, at 25c. a bunch. *Ans.* $72.

84. At 40c. a bunch for laths, 5c. a sq. yd. for labor, and 8c. a pound for nails, allowing a half-pound to a bunch, what will it cost to lath a hall 124 ft. long, 60 ft. wide, and 50 ft. high? *Ans.* $565.08.

85. How much more would be the cost if the laths in No. 84 crossed each other?

PLASTERING, PAINTING, CARPETING, ETC.

86. Find the cost of plastering the walls and ceiling of a room 87½ ft. long, 54 ft. wide, and 15 ft. high, at 21c. per sq. yd. *Ans.* $209.30.

87. A room is 40 ft. long, 25 ft. wide, and 14 ft. high. It has 2 doors, and 10 windows, each 12 ft. by 6 ft. What will it cost to plaster it, @ 18c. per sq. yd.?

NOTE.—Allow for only half the surface of the openings.

88. What will it cost to plaster 3 rooms, each of which is 18 ft. square, and 12 ft. high, @ 22½c. per sq. yd.?

89. At 16c. a sq. yd., find the cost of painting the walls in No. 88. *Ans.* $46.08.

90. What will it cost to kalsomine the ceilings in No. 88, @ 8½c. a sq. yd.? *Ans.* $9.18.

91. Find the cost of painting a double roof 56 ft. by 35 ft., @ 18c. a sq. yd. *Ans.* $78.40.

92. How many rolls of paper are required for the ceiling of a room 23 ft. long and 16 ft. wide?

NOTE.—A roll of wall-paper is 24 ft. long and 1½ ft. wide. A double roll is 48 ft. long. Dealers will not sell a part of a roll.

93. Find the number of strips of paper required for the walls of a room 15 ft. long, 12 ft. wide, and 12 ft. high, allowing for 4 windows and 1 door.

NOTE.—In estimating, paper-hangers divide the distance around the room, after deducting 3 ft. for each opening, by 1½ ft., the width of a roll.

94. How many rolls will paper a room 42 ft. long, 30 ft. wide, and 16 ft. high, with 10 windows and 4 doors? *Ans.* 46 rolls.

95. Find the cost of the paper in No. 94, at 65c. per double roll. *Ans.* $14.95.

96. In No. 94, how much would be gained by using single rolls, at 25c. per roll? *Ans.* $3.45.

97. Find the cost of papering a room 22 ft. long, 18 ft. wide, and 9 ft. high, with 5 openings and a cornice, at 50c. per double roll. *Ans.* $4.00.

NOTE.—An allowance of one foot is made for a cornice.

98. In No. 97, find the cost of a border, @ 8c. a yd.

99. At 15c. a roll, and 5c. a yd. for border, what will it cost to paper the walls of a hall 85 ft. long, 62 ft. wide, and 13 ft. high, with a cornice and 8 double openings?

100. A room is 26 ft. long and 18 ft. wide; how should the breadths of a carpet, one yard wide, be laid in order to have the least possible waste? *Ans.* 26 ft. long.

101. In No. 100, how many yd. would be required if the "breadths" were laid 18 ft. long? *Ans.* 54 yd.

102. A room is 35 ft. long, and 24 ft. wide; find the cost of carpeting it with carpet 27 in wide, @ 80c. a yd.

103. Find the cost of carpeting a room 16 ft. by 15 ft., with carpet 27 in. wide, at $1.25 a yard.

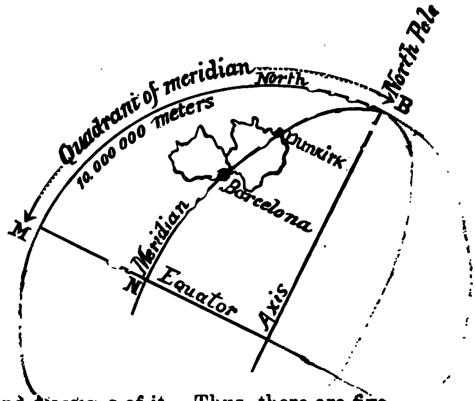
104. Allowing a waste of 6 in. per breadth in matching, find the cost of carpeting 4 rooms having the same dimensions as No. 103.

105. In No. 103, find the value of the carpet wasted by laying the breadths 15 ft. long.

METRIC SYSTEM

200. The ***Metric System*** is a series of measures whose units are derived from a fixed standard called the *Meter*.

201. The ***Meter*** is the *unit of length.* It is equal to the *ten-millionth* part of the quadrant of the terrestrial meridian, or 39.37 in.



This system of weights and measures is expressed on the decimal scale of notation. Having established the *unit* of each measure, we derive the other denominations by taking decimal multiples and divisions of it. Thus, there are five

PRINCIPAL UNITS.

The ***Meter*** (meter), for measures of length.
The ***Are*** (air), " " surface.
The ***Stere*** (stair), " " volume.
The ***Liter*** (leeter), " " capacity.
The ***Gram*** (gram), " " weight.

The higher denominations of each measure are expressed by placing, successively, before the name of its unit the following Greek words:

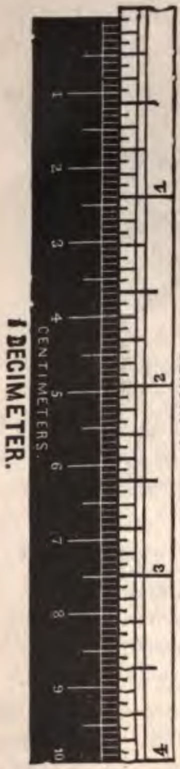
Deca	*Hecto*	*Kilo*	*Myria*
10,	100,	1000,	10,000,

The lower denominations, by the Latin prefixes,

Deci $\frac{1}{10}$, *Centi* $\frac{1}{100}$, *Milli* $\frac{1}{1000}$.

Hence, the formation of the tables may be illustrated by the following diagram:

	Length.	*Surface.*	*Volume.*	*Capacity.*	*Weight.*
Milli *Centi* *Deci* — *Deca* *Hecto* *Kilo* *Myria*	METER.	AR.	STERE.	LITER.	GRAM.



MEASURES OF LENGTH.

202. In measures of length the principal unit is the ***Meter.*** It is equivalent to 39.37 inches, or 1.0936 + yards. 1 yd. = .914392 meter.

TABLE.

10 millimeters (mm.)	= 1 centimeter ...	cm.
10 centimeters	= 1 decimeter.....	dm.
10 decimeters	= 1 ***meter***	m.
10 meters	= 1 decameter	Dm.
10 decameters	= 1 hectometer ...	Hm.
10 hectometers	= 1 kilometer.....	Km.
10 kilometers	= 1 myriameter...	Mm.

The meter is used for measuring short distances; as, the length of a street, the height of a house, etc.

For great distances, as, the length of a canal, a railroad, etc., the unit is the *kilometer* or the *myriameter*.

To show the correspondence between the Metric System of notan and that of United States money, the table for linear measure may be arranged as follows:

U. S. MONEY.......	Ten Thous.-dols.	Thous.-dollars.	Hundred-dollars.	Ten-dollars.	Dollars.	.	Dimes.	Cents.	Mills.
	Tens of thous.	Thousands.	Hundreds.	Tens.	Units.	|	Tenths.	Hundredths.	Thousandths.
METRIC SYSTEM....	5	6	4	2	8	.	3	1	7
	Mm.	Km.	Hm.	Dm.	M.	|	dm.	cm.	mm.

The unit, *meter*, corresponds to the unit, *dollar*, decimeter to dimes, centimeter to cents, etc.

The number is read 56428.317 *meters*. To express the same number in any other denomination, place the decimal point at the right of the required denomination. Thus, 564.28317 Hm. is read 564.28317 *hectometers;* 564283.17 dm., 564283.17 *decimeters.*

ORAL EXERCISES.

1. How many meters in a kilometer? In a hectometer? In a decameter?

2. How many decameters in a hectometer? In a myriameter?

3. What part of a meter is a decimeter? What part of a decimeter is a millimeter?

4. How many meters in 3 hectometers? In 5 kilometers?

5. How many mm. in 4 m.? In 6 m.? In 12 cm.?

6. How many m. in 100 dm.? In 2000 cm.?

7. Read each of the following: 6.5 dm.; 45 cm.; 251 Hm.; 12.64 Km.; 725.8 m.

8. Read 1283.14 m. in the different denominations that compose it.

WRITTEN EXERCISES.

1. Write 7 Dm. 5 m. 2 dm. in one number as meters.
Ans. 75.2 m.

In like manner, express:

2. 2 Km. 4 Hm. 5 Dm. 3 m. as decameters.
Ans. 245.3 Dm.

3. 8 Mm. 3 Km. 1 Hm. 4 dm. as meters.
Ans. 83100.4 m.

NOTE.—As in decimal notation, all vacant intervening orders are filled with ciphers.

4. 9 Hm. 3 dm. 8 mm. as meters. *Ans.* 900.308 m.

5. 3 Km. 7 Dm. 4 cm. as myriameters.
Ans. .307004 Mm.

6. Reduce 9.25 Km. to meters. *Ans.* 9250 m.

7. Reduce 1.2436 Hm. to decimeters.
Ans. 1243.6 dm.

8. Reduce 8956 m. to hectometers. *Ans.* 89.56 Hm.

9. Reduce 103.94 dm. to kilometers.
Ans. .010394 Km.

10. Reduce 342.75 Hm. to myriameters.
Ans. 3.4275 Mm.

11. In .208006 myriameters how many kilometers?

12. How many decimeters in 73.1804 hectometers?

13. In 89.725 myriameters how many kilometers?

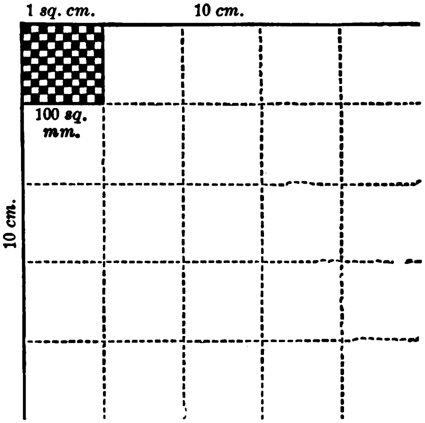
14. Reduce 4 Hm. 3 Dm. 2 cm. to myriameters.

MEASURES OF SURFACE.

203. In measures of surface the principal unit is the ***Square Meter.*** It is divided into 100 square decimeters, and is equal to 1.196 sq. yd., or 10.764 sq. ft.

The accompanying diagram represents one fourth of a square decimeter. One hundred such decimeters equal 1 square meter.

As indicated by the lines of division, a square decimeter contains 100 square centimeters, and each of the latter is equal to 100 sq. millimeters.



10 cm. × 10 cm. = 100 sq. cm. = 1 sq. dm.

10 dm. × 10 dm. = 100 sq. dm. = 1 sq. m.

TABLE.

100 sq. millimeters (sq. mm.) = 1 sq. centimeter.. sq. cm.
100 sq. centimeters = 1 sq. decimeter... sq. dm.
100 sq. decimeters = 1 ***sq. meter***..... sq. m.
100 sq. meters = 1 sq. decameter... sq. Dm.
100 sq. decameters = 1 sq. hectometer.. sq. Hm.
100 sq. hectometers = 1 sq. kilometer... sq. Km.

LAND MEASURE.

100 centares (ca.) = 1 ***are***.... .... a.
100 ares = 1 hectare .... Ha.

For surfaces of great extent, as the area of a country or state, the unit is the *square kilometer*.

In measuring land, the square meter is called the *centare;* the square decameter, an *are;* and the square hectometer, a *hectare.*

The standard unit in land measure is the *are.* For convenience, it is derived immediately from the decameter, instead of from the meter. It contains 100 sq. meters, or nearly 119.6 square yards.

ORAL EXERCISES.

1. How many sq. meters in a sq. decameter? In a sq. hectometer? In a sq. kilometer?

2. What part of a sq. centimeter is a sq. millimeter? What part of a sq. meter is a sq. decimeter?

3. How many centares in a hectare? Ares in a hectare?

4. What part of a hectare is an are? What part of an are is a centare?

WRITTEN EXERCISES.

1. Write 40 sq. Dm. 8 sq. m. as sq. meters.

SOLUTION.—As each denomination in square measure occupies two places, we must fill the vacant *tens'* place of sq. meters by a cipher. Hence, 40 sq. Dm. and 8 sq. m. equal 4008 sq. m.

2. Write 10 sq. Hm. 7 sq. Dm. 15 sq. m. 3 sq. dm. as square meters. *Ans.* 100715.03 sq. m.

3. Write 7 sq. Dm. 9 sq. m. 14 sq. dm. 5 sq. cm. as square centimeters. *Ans.* 7091405 sq. cm.

4. Write 5 Ha. 3 a. 4 ca. as ares. *Ans.* 503.04 a.

5. Express 540.32 a. as hectares. *Ans.* 5.4032 Ha.

NOTE.—When the *are* is the unit, move the point only as many places as indicated, but when the *square meter* is the unit, move it twice as many.

6. How many sq. meters in 8.23 hectares? *Ans.* 82300 sq. m.

7. How many sq. dm. in 72.05 sq. Dm.? *Ans.* 720500 sq. dm.

8. How many ares in 49630 ca.? *Ans.* 496.30 a.

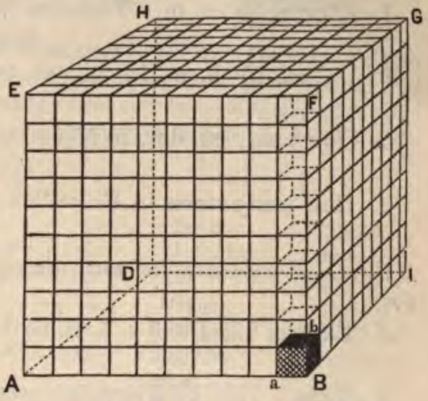
9. Change 6.1824 sq. kilometers to hectares.
Ans. 618.24 Ha.

10. Reduce 7585.40 hectares to centares.

MEASURES OF VOLUME.

204. In measures of volume the primary unit is the ***Cubic Meter.*** It is equal to 1.30799 cu. yd.

Let *ABCD-G* represent a cubic meter. Suppose it to be divided into ten equal parts of the same dimensions as the solid *ABCD.* Each of these parts is one meter square, and one decimeter thick, and contains one tenth of a cubic meter.



Secondly, each part may be subdivided into ten smaller solids, each of which being a decimeter square, and a meter long, will be .01 of a cubic meter.

Lastly, each of the smaller solids may also be divided into 10 cubic decimeters, such as *ab*, each being .001 of the cubic meter

TABLE.

1000 cu. mm. = 1 cu. cm. = .061 cu. in.
1000 cu. cm. = 1 cu. dm. = 1 liter (capacity) = .0353 cu. ft.
1000 cu. dm. = 1 ***cu. m.*** = 1 stere = 35.3165 cu. ft.

WOOD MEASURE.

10 decisteres (ds.) = 1 ***stere*** (s.) = 35.3163 cu. ft.
10 steres = 1 decastere (Ds.) = 2.759 cords.

In cubic measure the higher units are not used. The cubic decimeter and cubic centimeter serve chiefly as a basis for the measures of capacity and weight. When used to measure wood or stone, the cubic meter is called a *stere* (*stair*).

As 1000 units of a lower denomination, in this measure, make one unit of the next higher denomination, each denomination must have three places. Thus, 10 cu. m. 6 cu. dm. 47 cu. cm., written as cubic meters, is 10.006047 cu. m., and may be read 10 cu. m. and 6047 cu. cm.

WRITTEN EXERCISES.

1. Express 39 cu. m. 70 cu. dm. 2 cu. cm. as cu. meters.
Ans. 39.070002 cu. m.

2. Reduce 15 cu. m. 12 cu. cm. to cu. m.
Ans. 15.000012 cu. m.

3. How many cu. dm. in 56 cu. m. 25 cu. dm.?
Ans. 56025 cu. dm.

4. How many steres in 17 Ds. 9 s. 4 ds.?
Ans. 179.4 steres.

NOTE.—In wood measure, each denomination occupies only one place.

5. Express 235 Ds. 3 s. 5 ds. as decisteres.
Ans. 23535 ds.

6. Reduce 7186 decisteres to decasteres.
Ans. 71 Ds. 8 s. 6 ds.

7. Reduce 90357054 cu. cm. to cu. m.
Ans. 90.357054 cu. m.

8. Express 710.3486 cu. dm. as cu. meters.
Ans. .7103486 cu. m.

9. How many cu. dm. in 12 steres? In 6 cu. m.?

10. How many decasteres in 678 steres?

11. In 45.178527 cu. m. how many cu. cm.?

12. Reduce 11767.5 decisteres to decasteres.
Ans. 117.675 Ds.

MEASURES OF CAPACITY.

205. The unit of capacity is the ***Liter.*** It is equal in volume to 1 cu. decimeter, equal to 1.05671 qt. Liquid, or .9081 qt. Dry Measure.

The *liter* is equal to the capacity of a box, or cubical vessel, that is one tenth of a meter, or a decimeter, in length, breadth, and depth. The capacity of such a vessel may be divided into 1000 cubic centimeters, each of which, in measures of capacity, is called a *milliliter.*

TABLE.

		Dry M.	Liq. M.
10 milliliters = 1 centiliter	cl.	.61 cu. in.	.388 fl. oz.
10 centiliters = 1 deciliter.....	dl.	6.10 "	.845 gi.
10 deciliters = 1 ***liter***.... ...	l.	.908 qt.	1.0567 qt.
10 liters = 1 decaliter....	Dl.	9.081 qt.	2.64175 gal.
10 decaliters = 1 hectoliter....	Hl.	2.837 bu.	26.4175 "
10 hectoliters = 1 kiloliter	Kl.	28.37 bu. 1.308 cu. yd.	264.175 "
10 kiloliters = 1 myrialiter....	Ml.	283.72 bu.	2641.75 "

The *liter* is used in measuring liquids; but for measuring large quantities of grain, fruit, and liquids in casks, the hectoliter is employed. The latter is equal to 26½ gallons, or 2⅚ bushels.

ORAL EXERCISES.

1. How many liters in a hectoliter? How many decaliters in a kiloliter? In a hectoliter?

2. How many centiliters in a liter? How many milliliters in a liter?

3. What part of a liter is a deciliter? What part of a hectoliter is a liter?

4. How many liters in 4½ Dl.? How many kiloliters in 2000 l.?

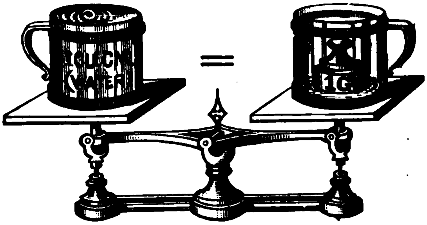
5. What part of a hectoliter is a dl.? What part of a myrialiter is a Kl.?

WRITTEN EXERCISES.

1. Express 9 Hl. 7 Dl. 4 l. 8 dl. as liters.
2. Express 15 Kl. 3 Dl. 6 dl. as hectoliters.
3. How many milliliters in 25.8 deciliters?
4. Change 718.12 l. to centiliters. *Ans.* 71812 cl.
5. Reduce 50637 l. to kiloliters. *Ans.* 50.637 Kl.
6. Write 123.4 dl. as hectoliters. *Ans.* .1234 Hl.
7. In 16 Dl. 7 dl. how many centiliters? *Ans.* 16070 cl.

MEASURES OF WEIGHT.

206. The unit of weight is the ***Gram.*** It is the weight of a cubic centimeter of distilled water at its maximum density, 39.2° F., or 4° C.; and is equal to .03527 oz. avoir., or 15.432 Troy gr.



A vessel having the capacity of a cubic decimeter, and filled with pure water at its greatest density, weighs one thousand grams, and is called a kilogram. A gram, therefore, is equal to the weight of the water contained in a small vessel having the capacity of a cubic centimeter.

TABLE.

10 milligrams	= 1 centigram...	cg.	.1543	gr. Tr.
10 centigrams	= 1 decigram....	dg.	1.543 +	"
10 decigrams	= 1 ***gram***......	g.	15.432 + .03527 +	" oz. av.
10 grams	= 1 decagram ...	Dg.	.3527 +	"
10 decagrams	= 1 hectogram...	Hg.	3.5274 +	"
10 hectograms	= 1 kilogram, or kilo ...	Kg	2.6792 2.2046	lb. Tr. lb. av.
10 kilograms	= 1 myriagram..	Mg.	22.046	"
100 kilos	= 1 quintal......	Q.	220.46 +	"
10 quintals	= 1 ***ton***........	T.	2204.62 +	"

The *kilogram* is used for weighing groceries and common articles; the *gram* for weighing letters, gold, and medicines. For articles of great weight, as coal and hay, the *metric ton* is the standard.

The *kilogram*, or *kilo*, weighs about 2¼ pounds, and the *metric ton* about $1\frac{1}{10}$ common tons.

NOTE.—The U. S. nickel five-cent piece is 2 centimeters in diameter, and weighs 5 grams. The lawful weight of a letter for a single postage of 2c., is 15 grams, or the weight of three nickels.

ORAL EXERCISES.

1. How many grams in a decagram? In a hectogram? In a kilogram?

2. How many decigrams in 40 centigrams? How many grams in 1000 milligrams?

3. What part of a decagram is a decigram? What part of a kilogram is a decagram?

4. How many metric tons in 3000 kilograms? How many hectograms in 400 grams?

5. How many grams in 75 decagrams? How many kilograms in 5 metric tons?

6. How many kilos in 4 quintals? How many tons in 40 quintals?

WRITTEN EXERCISES.

1. Express 16 Kg. 7 Dg. 8 g. as grams.

2. Write 45 Dg. 2 g. 9 dg. as grams. *Ans.* 452.9 g.

3. In 82.13 g., how many decigrams?

4. How many milligrams in 15 g. 4 dg.?

5. Change 837.52 cg. to grams. *Ans.* 8.3752 g.

6. Reduce 703.49 g. to hectograms.

7. How many metric tons in 9860.50 kilograms?
Ans. 9.8605 T.

WRITTEN REVIEW.

1. Find the sum of 71 m. 2.5 Hm. and 6 dm.
Ans. 321.6 m.

NOTE.—The ordinary operations in metric measures are performed according to the principles of decimals.

2. What is the difference between 127 Km. 3 Dm., and 274 Hm. 4 dm.? *Ans.* 99629.6 m.

3. Find the cost of 16.25 m. of cloth, at $2.50 per meter. *Ans.* $40.62½.

4. How much did I receive for 63.75 Dl. of milk, at 6 c. a liter? *Ans.* $38.25.

5. A man bought 36.42 hectares of land, at $1.25 per are. What was the cost of the land? *Ans.* $4552.50.

6. What is the value of a cubical bin of grain whose edge is 3 meters, at $1.25 per hectoliter?

7. A grocer bought 542 Kg. of raisins, at 20c. a kilo, and sold them at 25c. a pound. How much did he gain?

8. A room is 120 dm. long, and 75 dm. wide. How much will it cost to plaster the ceiling, at 22½c. per square meter? *Ans.* $20.25.

9. At 60c. a bu., what will 80 hectoliters of corn cost?
Ans. $136.176.

10. If I buy 130 yd. of carpet, at 85c. a meter, and sell it at the same price per yard, how much will I gain?
Ans. $9.46.

11. What will be the gain on 200 steres of wood, bought at 60c. a stere, and sold at $2.50 a cord?
Ans. $17.95.

12. How many cubic meters in a block of marble 11 ft. long, 5.351 ft. wide, and 3 ft. thick? *Ans.* 5 cu. m.

13. What must be the height of a pile of wood which is 8.5 m. long, 6.3 m. wide, and contains 289.17 steres?
Ans. 5.4 m.

14. How many 5c. pieces can be made of a bar of metal weighing 86 hectograms? *Ans.* 1720 pieces.

15. What will be the profit on 2150 liters of molasses, bought at 4½c. per liter, and sold at 28c. a gallon?

16. How many miles will an engine run in 8 hours, at the rate of 28.5 Km. an hour? *Ans.* 141.68 miles.

17. From a barrel, containing 76 Kg. of sugar, were taken three parcels of 42.50 Hg. each, and four parcels of 97.50 Hg. each. How much sugar was left in the barrel?
Ans. 24.25 Kg.

18. How many jets can be fed by a gasometer holding 36000 cu. m., if each jet burn 150 l. in an hour, and is used for 6 hours every evening? *Ans.* 40000 jets.

19. Find the cost of papering a room 13 m. square, and 2.5 m. high, with paper 20 cm. wide, and 6.25 m. in a roll, at 80c. a roll? *Ans.* $83.20.

20. A lady bought 66.1628 yd. of silk in New York. What would she have paid for it in Paris, at $5¼ per meter? *Ans.* $317.63.

PERCENTAGE.

1. ***Percentage*** is a method of computation having 100 for its unit or base.

The sign of percentage is the symbol, %, called per cent., which is an abbreviation of the Latin per centum, and signifies by the hundred; thus 8% means 8 of every hundred. It may refer to any denomination, and may denote 8 cents of every hundred cents, 8 dollars of every hundred dollars, 8 bushels of every hundred bushels, etc.

2. The elements of percentage are the ***Base,*** the ***Rate,*** and the ***Percentage.***

3. The ***Base*** is the number of which the percentage is taken.

4. The ***Rate*** is the number denoting how many hundredths of the base are to be taken.

5. The ***Percentage*** is the result obtained by taking a certain number of hundredths of the base.

In the statement 8% of $400 is $32, the base is $400 ; the rate, 8%, or $\frac{2}{25}$, and the percentage $32.

6. One per cent. of any number is one hundredth of it; four per cent. is four hundredths, and, in general, any per cent. of a number may be expressed as so many decimal or fractional parts of that number; thus:

RATE.		DECIMAL.		FRACTIONAL.	RATE.		DECIMAL.		FRACTIONAL.
1%	=	.01	=	$\frac{1}{100}$	$\frac{1}{2}$%	=	.005	=	$\frac{1}{200}$
5%	=	.05	=	$\frac{1}{20}$	$\frac{3}{4}$%	=	.0075	=	$\frac{3}{400}$
10%	=	.10	=	$\frac{1}{10}$	50%	=	.50	=	$\frac{1}{2}$
12$\frac{1}{2}$%	=	.12$\frac{1}{2}$	=	$\frac{1}{8}$	66$\frac{2}{3}$%	=	.66$\frac{2}{3}$	=	$\frac{2}{3}$
25%	=	.25	=	$\frac{1}{4}$	75%	=	.75	=	$\frac{3}{4}$
33$\frac{1}{3}$%	=	.33$\frac{1}{3}$	=	$\frac{1}{3}$	125%	=	1.25	=	1$\frac{1}{4}$

WRITTEN EXERCISES.

7. The base and rate being given, to find the percentage.

1. A man having $140, paid out 35% of it for provisions; how much did he pay out?

OPERATION.

$140 × $\frac{7}{20}$ = $49.

SOLUTION.—Since 35% = $\frac{35}{100}$ or $\frac{7}{20}$, then 35% of $140 = $\frac{7}{20}$ of $140, or $49.

RULE.—*To find the percentage, multiply the base by the rate.*

What is:

2. 15% of 640 acres? *Ans.* 96 acres.

3. 9% of 200 gallons? *Ans.* 18 gallons.

4. 36% of 50 men? *Ans.* 18 men.

5. 77% of 500 miles? *Ans.* 385 miles.

6. 25% of 740 bushels? *Ans.* 185 bushels.

7. 48% of 175 pounds? *Ans.* 84 pounds.

8. 225% of 996 tons? *Ans.* 2241 tons.

9. 524% of 1250 sheep? *Ans.* 6550 sheep.

10. 45% of 1 mile? *Ans.* 144 rods.

11. A man having $1960, spent 15% of it. How much had he left? *Ans.* $1666.

12. A trader having $1650, spent 13⅓% of it for coffee, at 16c. a pound. How many pounds did he buy? *Ans.* 1375 pounds.

13. Having a farm of 240 acres, I sold 43¾% of it for $7875. What price per acre did I receive? *Ans.* $75.

14. A merchant having $10400 in bank, drew out 18¾% of it to buy wool at 30c. a pound. How many pounds did he buy? *Ans.* 6500 pounds.

15. A speculator having $5820, invested 13⅓% of it in horses, and 12½% of the remainder in cattle. How much money had he left? *Ans.* $4413½.

PROFIT AND LOSS.

8. ***Profit and Loss*** are terms indicating the sum gained or lost in business transactions.

9. The quantities in Profit and Loss are the *Cost*, the *Rate*, the *Gain* or *Loss*, and the *Selling Price*, corresponding to the base, the rate, the percentage, and the amount or difference in percentage.

WRITTEN EXERCISES.

10. To find the gain, the loss, or selling price, when the cost and the rate are given.

1. Bought goods to the amount of $4000, and sold them so as to gain 30% on the cost; what was my gain?

OPERATION.

$\frac{3}{10}$ of \$4000 = \$1200

SOLUTION.—The goods were bought for $4000, and sold at a gain of 30%, or $\frac{3}{10}$ of \$4000, which is \$1200.

2. I bought a horse for $375, and sold it at a loss of $16\frac{2}{3}\%$. What was my loss? *Ans.* $\$62\frac{1}{2}$.

3. Paid $240 for a carriage, and sold it at a profit of $31\frac{1}{4}\%$. What did I make? *Ans.* $75.

4. I paid 7c. a pound for 2480 pounds of coffee, and sold it at a loss of 10% on the cost. What was my loss? *Ans.* $17.36.

5. A farm cost me $2475; I sold it at a loss of $6\frac{2}{3}\%$. How much did I get for it? *Ans.* $2310.

6. A man bought 100 barrels of flour for $500, and sold it so as to gain 20% of the cost. What did he receive per barrel? *Ans.* $6.

7. A shoe dealer marks his goods at an advance of 20% of the cost. How should a pair of shoes be marked that cost $2.50? *Ans.* $3.

8. At what price must an article that cost $100 be sold to gain 20%? 25%? 40%? 50%? 100%? 150%? 200%? 250%?

9. Some cloth bought at $8 a yard was sold at 50% advance. What was the gain?

10. A merchant bought two horses for $125 each, and, in selling them, gained 24% on one, and lost 24% on the other. What did he gain or lose by the sale?

11. In a school of 480 pupils, 43¾% were boys. How many girls were there? *Ans.* 270.

12. A drover had 160 calves, which he sold at $7.50 a head, and then spent 55% of the money for sheep, at $2.75 each. How many sheep did he buy? *Ans.* 240 sheep.

13. Bought 37 cows for $888, and sold them at a profit of 15%. Find the selling price per head. *Ans.* $27.60.

14. A farmer having $2400, paid 18¾% of it for sheep, 24% of the remainder for horses, and the rest for cows, at $78 each. How many cows did he buy? *Ans.* 19 cows.

15. Sold a farm of 120 acres, at $62.50 an acre, and invested 66% of the proceeds in lots, at $330 each. How many lots did I buy? *Ans.* 15 lots.

16. Having raised 1980 bushels of potatoes, I sold 41⅔% of them at 48c. a bushel. How much did I get for the quantity sold? *Ans.* $396.

COMMISSION.

11. ***Commission*** is an allowance made an agent for transacting business.

12. An ***Agent*** is a person who transacts business for another. He is also called *Commission Merchant, Factor,* or *Broker.*

13. To find the commission, or net proceeds, when the base and the rate are given.

1. What is the commission, at 6⅔%, on the sale of 240 bushels of wheat at 75 cents a bushel? The proceeds?

OPERATION.

$240 \times .75 = \$180$

$\$180 \times \frac{1}{15} = \$12$ commission

$\$180 \times (1.00 - .06\frac{2}{3}) = \$168$ proceeds.

SOLUTION. — The value of 240 bushels at 75 cents a bushel = $180. And the commission = $\frac{1}{15}$ times $180 = $12. Hence the commission is $12. To find the proceeds, we multiply the base $180, by 100% minus the rate and obtain $168.

RULE.—*I. To find the commission, multiply the base by the rate.*

II. To find the proceeds, multiply the base by 100% *minus the rate.*

2. An agent sold a house for $3960, at 2⅓% commission, what was the commission? *Ans.* $92.40.

3. A commission merchant sold 128 bbl. of sugar, each weighing 240 lb., at 8⅓ cents a pound. Find his commission at 2¾%. *Ans.* $70.40.

4. An agent sold 260 bbl. of sugar, each weighing 240 lb., at 8¾ cents a pound; and 185 bbl. of molasses, each containing 48 gal., at 62½ cents a gallon. Find his entire commission at 2%. *Ans.* $220.45.

5. An agent sold a farm of 80 acres, at $120 per acre; what was his commission at ¾%? *Ans.* $72.

6. If I charge 2½% for collecting $1300, what amount must I pay over to my employer?

7. An agent sells a consignment of oats for $1528; he pays $12.50 for freight, etc., and charges 1¾% commission; find the net proceeds. *Ans.* $1488.76.

INTEREST.

14. ***Interest*** is a sum paid for the use of money.

15. ***Principal*** is the sum for the use of which interest is paid.

16. ***Rate*** is a certain per cent. of the principal which indicates the interest on one dollar for one year.

17. ***Time*** is the period during which money is on interest.

18. ***Amount*** is the sum of the principal and the interest.

19. ***Simple Interest*** is the sum charged for the use of the principal.

WRITTEN EXERCISES.

1. What is the interest of $5000 for 9 years, 3 months, 9 days, at 8%? Amount?

OPERATION.

$5000
.08
$400.00 Int. for 1 yr.
$9\frac{11}{40}$
11000
360000
$3710.00 Int. for 9 yr. 3 mo. 9 da.
$5000.00
$8710.00 Amount.

SOLUTION. — Reducing the 3 mo. and 9 da. to a fraction of a year, we obtain $\frac{11}{40}$. The interest of $5000, at 8% for 1 yr., equals .08 times $5000, or $400. Hence the interest of $5000 for $9\frac{11}{40}$ years, equals $9\frac{11}{40}$ times $400 = $3700. Adding the principal, $5000, to the interest for the given time, we get $8710 for the amount.

RULE.—*Multiply the principal by the rate and that product by the time expressed in years. The result will be the required interest.*

Find the interest of: Amount of:

2. $180.50 for 2 yr. 8 mo., at 8%. *Ans.* $38,506.

3. $400 for 1 yr. 9 mo., at 9%. *Ans.* $63.

4. $175 for 4 yr. 2 mo., at 7%. *Ans.* $51.04.

5. $240.20 for 1 yr. 7 mo., at 5%. *Ans.* $19.02.

6. $1000 for 3 yr. 7 mo., at 6%. *Ans.* $215

7. $26.84 for 2 yr. 6 mo., at 5%.

8. $950 for 2 yr. 7 mo., at 10%. *Ans.* $245.42.

9. $196 for 5 yr. 7 mo., at 9%. *Ans.* $98.49.

10. $95.40 for 3 yr. 9 mo., at 8%. *Ans.* $28.62.

11. $240.80 for 5 yr. 10 mo., at 6%. *Ans.* $84.28.

12. $340.60 for 4 yr. 5 mo., at 6%. *Ans.* $90.259.

13. $500 for 2 yr. 9 mo., at 8%. *Ans.* $110.

14. $26.84 for 2 yr. 6 mo., at 5%.

15. $200.25 for 3 yr. 5 mo., at 8%. *Ans.* $54.74.

16. $76.50 for 2 yr. 2 mo., at 5%. $8.29.

17. $847.25 for 1 yr. 8 mo. 7 da., at 8%. *Ans.* $114.28.

18. $269.50 for 2 yr. 7 mo. 10 da., at 5%. *Ans.* $35.18.

19. $280.75 for 3 yr. 2 mo. 12 da., at 7%. *Ans.* $62.89.

SIX PER CENT. METHOD.

1. What is the interest of $180 for 6 yr. 10 mo. 12 da., at 6%?

OPERATION.

Int. of $1 for 6 yr. $= .06 \times 6 =$ $0.36

" " " " 10 mo. $= .00\frac{1}{2} \times 10 =$ .05

" " " " 12 da. $= .000\frac{1}{6} \times 12 =$ .002

" " " " 6 yr. 10 mo. 12 da. = $0.412

" " $180 " 6 yr. 10 mo. 12 da. = $.412 \times 180$

$.412 \times 180 =$ $74.16

SOLUTION.—The interest on $1 for 1 yr., at 6%, is 6c. For 6 yr. it is 6 times 6c., or 36c. For 1 month, which is $\frac{1}{12}$ of a year, it is $\frac{1}{12}$ of 6c., or $\frac{1}{2}$ of a cent. Hence for 10 mo. it is 10 times $\frac{1}{2}$c., or .05. Finally, since the interest on $1 for 1 mo. is $\frac{1}{2}$c., or 5 mills, for 1 day it is $\frac{1}{30}$ of 5 mills, or $\frac{1}{6}$ of a mill; and, therefore, for 12 days it is 12 times $\frac{1}{6}$ of a mill, or .002. Thus we find the interest of $1 for 6 yr. 10 mo. 12 da., to be $0.412. Multiplying the interest on $1 for the given time, by $180, the principal, we obtain $74.16, the required interest.

RULE.—*I. Multiply the rate, 6%, by the number of years, take $\frac{1}{2}$ of the number of months as cents, and $\frac{1}{6}$ of the number of days as mills.*

II. Add these partial interests for the interest of $1 for the given time at 6%.

III. Multiply the sum thus obtained by the principal, and the product will be the required interest.

IV. For any other rate, first find the interest at 6%, and then take as many sixths of it as that rate is of six.

WRITTEN EXERCISES.

What is the interest of:

2. $440 for 2 yr. 5 mo. 15 da., at 6%? *Ans.* $64.90.

3. $455 for 3 yr. 4 mo. 6 da., at 4%? *Ans.* $60.97.

4. $650 for 2 yr. 8 mo. 12 da., at 7%? *Ans.* $122.85.

5. $420 for 2 yr. 7 mo. 15 da., at 3$\frac{1}{3}$%? *Ans.* $36.75.

6. $725 for 3 yr. 1 mo. 18 da., at 9%? *Ans.* $204.45.

7. $930 for 11 mo. 10 da., at 6%? *Ans.* $52.70.

8. $45 for 3 yr. 3 mo. 6 da., at 4$\frac{1}{2}$%? *Ans.* $66.15.

9. $580 for 81 da., at 6%? *Ans.* $7.83.

10. $2160 for 1 yr. 5 da., at 6%? *Ans.* $131.40.

What is the amount of:

11. $8.50 for 1 yr. 9 mo. 12 da., at 6%?
Ans. $9.41.

12. $495.95 for 5 yr. 5 mo. 5 da., at 6¾%?
Ans. $677.74.

13. $746.25 for 1 yr. 10 mo. 12 da., at 5%?
Ans. $815.90.

14. $650 for 3 yr. 10 mo. 21 da., at 7%?
Ans. $827.07.

15. $0.05 for 20 yr. 10 mo. 15 da., at 8%?
Ans. $0.1335.

16. $1024 for 1 yr. 7 mo. 24 da., at 3⅓%?
Ans. $1080.32.

17. $1125 for 3 yr. 9 mo. 18 da., at 5%?
Ans. $1338.75.

18. $1250 for 3 yr. 7 mo. 6 da., at 6½%?
Ans. $1542.50.

19. $1325 for 1 yr. 10 mo. 12 da., at 6¾%?
Ans. $1491.95.

20. $1850 for 3 yr. 3 mo. 18 da., at 3⅓%?
Ans. $2053.50.

21. $2275 for 5 mo. 21 da., at 8%? *Ans.* $2361.45.

22. $1860 for 10 mo. 23 da., at 6%? *Ans.* $1960.13.

23. $2160 for 1 yr. 5 da., at 6%? *Ans.* $2291.40.

24. What is the interest on $1200 from July 6, 1896, to Oct. 10, 1899, at 7%? *Ans.* $273.93.

25. What is the interest on $420 from March 3, 1897, to Sept. 14, 1900, at 6%? *Ans.* $88.97.

26. What is the interest, at 6%, on $964.40 from Nov. 18, 1896, to Feb. 6, 1899? *Ans.* $128.27.

27. What is the interest of $983.27 from April 9, 1897, to Jan. 13, 1900, at 8%? *Ans.* $217.193.

28. Find the interest on $371.48 from July 11, 1900, to Oct. 18, 1911, at 12%. *Ans.* $502.36.

29. What is the amount of $137.50 from July 3, 1898, to Feb. 3, 1899, at 9%? *Ans.* $144.72.

30. What is the amount of $153.80 from June 10 to Dec. 31, at 5%? *Ans.* $158.09.

31. What is the amount of $234.60 from Aug. 2, 1901, to March 8, 1902, at 5½%? *Ans.* $241.99.

32. Mr. Jones paid a debt of $1560, which had been at interest for 7 mo. 13 da., at 6%. What was the amount paid? *Ans.* $1617.98.

33. My note, given 2 yr. 10 mo. 16 da. ago, for $3180, and bearing 4% interest, is due to-day. What is the amount due? *Ans.* $3546.05.

34. On the 15th day of Aug., 1896, I borrowed $2500 at 7% interest. Find the total amount due Aug. 15, 1898. *Ans.* $2850.

35. If I borrow $930 at 6%, what will be due at the end of 11 mo. 10 da.? *Ans.* $982.70.

36. I owe a merchant $840 and agree to pay him the debt in 44 days, with interest at 6%; how much shall I have to pay him? *Ans.* $846.16.

37. A man invested $1150 for 2 yr. 4 mo. 24 da., at 5½%. What were the interest and the amount? *Ans.* $1301.80 amount.

38. Gave a note of $793.26 Apr. 6, 1890, on interest at 7%. What is due Sept. 10, 1892? *Ans.* $928.07.

39. What is the interest on a note of $264.42, given Jan. 1, 1892, and due Oct. 10, 1895, at 4%? *Ans.* $39.93.

40. Mr. Ray paid a debt of $840 which had been at interest 10 mo. 18 da., at 6%. What was the amount of the debt?

EXAMINATION PAPERS.

1.

1. Find the prime factors of 2964.

Ans. 2 × 2 × 3 × 13 × 19.

2. Find the smallest number that is exactly divisible by each of the nine digits. *Ans.* 2520.

Find the G. C. D. of the following:

3. 476, 744, 1148. *Ans.* 4.

4. 462, 882, 546. *Ans.* 42.

5. 1073, 1537, 1160. *Ans.* 29.

6. 12032, 16403. *Ans.* 47.

Find the L. C. M. of the following:

7. 153, 204, 510. *Ans.* 3060.

8. 84, 126, 540. *Ans.* 3780.

9. 126, 154, 280. *Ans.* 27720.

10. 291, 485, 679. *Ans.* 10185.

11. Find the G. C. D. and the L. C. M. of 492, 744, 1044. *Ans.* 12. 2653848.

12. What are the prime factors of each of the following numbers: 42, 48, 126, 144? Indicate the combination of factors necessary to produce (*a*) the G. C. D., (*b*) their L. C. M.

Ans. G. C. D. 2 × 3. L. C. M. 2 × 3 × 7 × 2 × 2 × 2 × 3.

13. Find the prime factors of 1226, 1938 and 2346. Indicate which of these factors must be combined to produce (*a*) the G. C. D., (*b*) the L. C. M.

Ans. G. C. D. 2. L. C. M. $2\times3\times613\times17\times19\times23$.

2.

Reduce the following fractions to their simplest form:

1. $\frac{3335}{4669}$. *Ans.* $\frac{5}{7}$. *2.* $\frac{1683}{2574}$. *Ans.* $\frac{17}{26}$. *3.* $\frac{54520}{72239}$. *Ans.* $\frac{40}{53}$.

4. $\frac{4\frac{3}{8}-1\frac{3}{4}}{4\frac{2}{7}\times5\frac{3}{5}}$. *5.* $\frac{363.25}{482\frac{2}{3}}+\frac{5}{9}$. *6.* $\frac{17\frac{5}{12}-9\frac{3}{4}+4\frac{5}{7}}{\frac{5}{9}\times9\frac{2}{7}}$.

7. $\frac{4\frac{1}{2}+2\frac{2}{3}}{4\frac{1}{2}\times2\frac{2}{3}}\times72$. *8.* $\left(\frac{20}{39}\times\frac{52}{64}+3\frac{5}{6}\right)\times\frac{3}{17}-\left(.75\div\frac{15}{4}\right)$.

9. $\frac{\frac{2}{3}\times\frac{4}{9}-\frac{3}{5}\times\frac{1}{7}}{4\frac{1}{2}-1\frac{2}{3}+2\frac{1}{3}}$. *10.* $\frac{1\frac{1}{2}+2\frac{3}{5}}{4\frac{1}{10}-2\frac{4}{5}}\times\frac{\frac{3}{4}\times1\frac{2}{3}}{1\frac{1}{4}}\times6\frac{1}{2}$.

11. $\frac{4\frac{3}{8}\times\frac{9}{14}+1.8}{8\frac{4}{7}\div\frac{21}{40}}$. *12.* $\frac{4\frac{2}{3}+5\frac{1}{2}}{9\frac{3}{4}-4\frac{2}{3}}\times\frac{\frac{3}{5}\times2\frac{1}{6}}{130}$ *Ans.* $\frac{1}{50}$.

13. $\frac{3\frac{3}{4}-\frac{2}{7}\times4.2}{\frac{5}{18}\div\frac{20}{27}}$ *14.* $\frac{5\frac{5}{8}\times\frac{12}{27}\div2.5}{3\frac{3}{26}\div\frac{26}{39}\times\frac{16}{45}}+2.75-\frac{4}{9}$. *Ans.* $2\frac{49}{54}$.

15. $\frac{(1\frac{2}{3}+1\frac{3}{4})\times3}{\frac{7}{8}\times2\frac{1}{2}-\frac{4}{5}\div\frac{3}{7}}$. *16.* $\frac{\frac{3}{4}+2\frac{7}{8}}{\frac{4}{5}\times\frac{3}{4}}-\frac{\frac{2}{3}\times\frac{5}{6}}{4\frac{1}{2}+1\frac{3}{4}}$.

17. $\frac{(\frac{3}{5}+\frac{2}{7})(\frac{2}{3}-\frac{1}{4})}{2\frac{1}{2}\times\frac{7}{8}+4\frac{1}{3}-2}$. *18.* $\frac{25}{18}\times\frac{1\frac{13}{15}}{2\frac{16}{27}}\div\frac{4.375\div\frac{7}{4}}{5\frac{3}{4}-\frac{11}{3}}$. *Ans.* $\frac{5}{6}$.

19. $\frac{1}{3}$ of $\frac{3\frac{1}{7}}{4}$ of $2\frac{1}{3}\times14$. *20.* $\left\{\left(\frac{28}{45}\times2\frac{1}{7}\right)+\left(\frac{51}{36}\div4\frac{13}{18}\right)\right\}\div6.3$.

21. $\frac{\frac{11}{27}\div(\frac{13}{9}+2\frac{5}{6})}{.25\times\frac{4}{7}}$. *22.* $\frac{\frac{1}{5}\times2\frac{1}{7}}{\frac{1}{2}+\frac{3}{8}}\div\frac{\frac{3}{5}+\frac{1}{3}}{\frac{16}{5}-2\frac{1}{2}}$. *Ans.* $\frac{18}{49}$.

Simplify and reduce the result of the following fractions to a decimal and express it in words:

23. $\dfrac{18\frac{1}{2} \times 7\frac{2}{5}}{8\frac{3}{4} - 6\frac{1}{5}}$ *24.* $\dfrac{41871}{111656}$. *25.* $\dfrac{\frac{3}{5} \times 1.25}{5\frac{3}{4} - 4.25}$. *Ans.* 53.68 +.

26. $\dfrac{2\frac{1}{3} + 4\frac{1}{4}}{4\frac{1}{6}}$ *27.* $\left\{\left(14\frac{2}{7} \div \frac{15}{56}\right) - \left(6\frac{3}{8} \times \frac{32}{17}\right)\right\} \times .0625.$

3.

1. Express in Roman notation 1492 and 1893. Express in words the sum of 609.0031 and 394.048.

2. Write in Roman notation 1894, 10486. Express in words 10010.0010, 4268.5, 185,851.

3. Write in Arabic notation eight hundred eight thousand eighty-eight and eighteen-thousandths. Express in words 61056.003. Write in Roman notation 1861.

Ans. 808088.018.

4. Write in Arabic notation four million seventeen thousand eighty-three and two hundred six hundred-thousandths. Write in Roman notation 1898.

Ans. 4017083.00206.

5. Reduce $\frac{7}{20}$, $\frac{11}{16}$, and $\frac{18}{125}$ to decimals; from the sum of these decimals subtract .4315 and express the result as a common fraction. *Ans.* $\frac{3}{4}$.

6. Reduce $\frac{128}{125}$, $\frac{200}{512}$, and $\frac{250}{320}$ to decimals. Add these decimals and express their sum as a common fraction in its simplest form. *Ans.* $2\frac{1567}{8000}$.

7. Reduce $\frac{19}{3040}$ and $\frac{3}{8}$ to decimals and divide the first by the second. *Ans.* .016.

8. Divide 15 millionths by 50 and express answer in words. *Ans.* .0000003.

9. Divide one millionth by eight ten-thousandths and express the result in words. *Ans.* .00125.

10. Multiply four millionths by six ten-thousands, divide the result by twelve and express the quotient in words. *Ans.* .0000000002.

11. Divide the sum of 18 thousandths, 106 ten thousandths, 84 hundreths, and 509 ten-thousandths by 15 millionths. *Ans.* 61300.

12. Multiply two thousand nine ten-millionths by thirty thousand two and seven-tenths, and divide the product by three ten-thousands. *Ans.* 20091.8081.

13. Multiply 65.15 by 3.14159 and divide the result by 57.296, finding a result correct to three decimal places. *Ans.* 3.572 +.

4.

Solve the first 5 problems by analysis.

1. If $\frac{2}{3}$ of an article costs $\frac{5}{8}$ of a dollar, what will $\frac{3}{5}$ of it cost? *Ans.* \$$\frac{9}{12}$.

2. If $\frac{4}{5}$ of an article cost $\frac{7}{8}$ of a dollar, how much will $\frac{8}{14}$ of it cost? *Ans.* \$$\frac{5}{8}$.

3. A railway train runs $\frac{2}{3}$ of a mile in $\frac{4}{5}$ of a minute; find the velocity per hour. *Ans.* 50 miles.

4. A man walks $8\frac{3}{4}$ miles in 2 hr. 20 min.; how long will it take him to walk $11\frac{1}{8}$ miles? *Ans.* 2 hr. 58 min.

5. A yardstick perpendicular to a level floor casts a shadow 28 in. long; find the height of a flagstaff which at the same time casts a shadow 70 ft. long. *Ans.* 90 ft.

6. The owner of $\frac{3}{11}$ of a mine sold $\frac{9}{10}$ of his share for \$40500. What should he who owns $\frac{3}{5}$ of a mine get for $\frac{5}{9}$ of his share? *Ans.* \$55000.

7. A farmer owns 356½ acres of land, sells ¼ of it to one man and ⅖ of it to another; what is the value of the remainder at $40 an acre? *Ans.* $4991.

8. A and B together have $100 and ⅗ of A's money is equal to $\frac{9}{10}$ of B's money; how much has each?
Ans. $60 *A*'s, $40 B's.

7

9. A, B, and C together have $250; B has ⅔ as much as A, and C has ¼ as much as A and B together. How much has each? *Ans.* $50, $80, $120.

10. A and B together have $70; C has twice as much as B, and A has three times as much as C. How much has each? *Ans.* B $10, C $20, A $60.

11. A, B, and C together have $200; twice B's money is equal to three times C's money, and B and C together have ⅔ as much as A. How much has each?
Ans. $32, $48, $120.

12. A, B, and C together have $160; twice A's money is equal to half of B's money, and C has ⅗ as much as A and B together. How much has each? *Ans.* $20, $80, $60.

13. A has twice as much money as B, and C's money is equal to the sum of one-third A's money and one-half B's money; the three together have $25. How much has each?
Ans. A $12, B $6, C $7.

14. A can dig a ditch in 6 days, B in 8 days, and C in 12 days. How long will it take them to do the work if all work together? *Ans.* 2⅔ days.

15. A can do a piece of work in 4 days, B can do it in 6 days, and C in 12 days. How many days will it take all three working together? *Ans.* 2 days.

16. A can do a piece of work in 10 days; A and B together can do the same work in 7 days. In how many days can B do it working alone? *Ans.* 23⅓ days.

17. A farmer plants corn on $\frac{1}{8}$ of his land, potatoes on $2\frac{1}{2}$ times as much, and sows the remainder with wheat; he sells the wheat at 60 cents a bushel, and receives for it $180. If the yield of wheat was 20 bushels an acre, how much land had he? *Ans.* 50 acres.

5.

1. Find the cost of a stick of timber 40 ft. long, 12 in. wide, and 9 in. thick, at $12.50 per M., board measure. *Ans.* $$4\frac{1}{2}$.

2. Find the cost, at $17.50 a 1000 ft., board measure, of 35 3 in. planks each 22 ft. long and 16 in. wide. *Ans.* $53.90.

3. What is the cost, at $18 a thousand ft., of lumber for a floor 21 ft. long by 16 ft. wide, allowing $\frac{1}{8}$ of the lumber for matching? *Ans.* $5.292.

4. What is the cost, at $14 a 1000 ft., board measure, of 6 pieces of timber, each 24 ft. long, 10 in. wide, 8 in. thick? *Ans.* $13.44.

5. Find the cost of 8 sticks of timber, each 42 ft. long, 10 in. wide, 8 in. thick, at $18 per M., board measure. *Ans.* $40.32.

6. What is the cost of 4 sticks of timber, each 8 in. by 10 in. and 30 ft. long, at $15 a 1000 ft.? *Ans.* $12.

7. Find the cost of 5 sticks of timber, each 20 ft. long and 6 in. by 9 in., at $16 a 1000 ft. *Ans.* $7.20.

8. Find the cost of the following bills of lumber:

78 boards 13′ × 16″ × $\frac{7}{8}$″ at $16.50 a 1000 ft.
18 joists 10′ × 4″ × 3″ at $13.75 a 1000 ft.

Ans. $24.75.

9. 6 pieces 16′ × 8″ × 2″ at $14.50 per M.
8 “ 12′ × 3″ × 4″ at $15.25 “ “
16 “ 12′ × 1″ × 8″ at $16.10 “ “
Ans. $5.40.

10. 3 pieces 8″ × 6″ × 12′ at $17 a 1000 feet.
30 “ 12″ × 2″ × 14′ “ $20 “ “
20 “ 10″ × ⅞″ × 16′ “ $25 “ “
Ans. $25.57.

11. 20 pieces 13′ × 6″ × 10″ at $14 a 1000 feet.
10 “ 16′ × 2″ × 4″ “ $16 “ “
6 “ 24′ × 8″ × 10″ “ $15 “ “
Ans. $34.31.

6.

1. Find the cost of carpeting a room 12 ft. × 16 ft. with material ¾ of a yd. wide at 60 cents a yard.
Ans. $17.06⅔.

2. What is the cost of carpeting a room 18 ft. long and 12 ft. wide with carpet ¾ of a yd. wide, at $1.25 a yard?
Ans. $40.

3. How many cords of wood can be stored in a shed 16 ft. long, 12 ft. wide, and 6 ft. high?
Ans. 9 cords.

4. Find the cost of digging a ditch 20 rd. long, 4½ ft. deep, and 18 in. wide, at 25 cents a cubic yard of excavations.
Ans. $20.62½.

5. Bought 1½ acres of land at $3200 an acre and sold it at 12 cents a square foot; find the entire gain.
Ans $3040.80.

6. Find the cost of digging a cellar 60 ft. by 30 ft. and 8 ft. deep, if the excavating costs 50 cents a cubic yard.
Ans. $266.66.

7. Find in bushels the capacity of a bin 8 ft. long, 5 ft. deep, and 4 ft. wide. *Ans.* 128$\frac{4}{7}$ bushels.

8. What is the contents in bushels of a bin 8 ft. long, 4 ft. wide, 6 ft. deep? *Ans.* 154$\frac{2}{7}$ bushels.

9. Find the entire surface of the four walls and ceiling of a room 18 ft. long, 14 ft. wide, and 9 ft. high.
Ans. 828 sq. ft.

10. A pile of wood is 4 ft. wide, 10 ft. high, and 20 yards long. What is the value, at $5 a cord? *Ans.* $93.75.

11. How much lumber will be required to cover the four walls of a room 16 ft. by 18 ft. and 10 ft. high, and how much will the lumber cost at $16 per M.?
Ans. $10.88.

12. Find the cost, at 12 cents a square yd., of plastering the four walls and ceiling of a room 14 ft. by 12 ft. and 9 ft. high, allowing 15 sq. yd. for doors and windows.
Ans. $6.68.

13. What is the cost of plastering the walls and ceiling of a room 14 ft. by 10 ft. and 9 ft. high, at 35 cents a sq. yd., allowing 8 sq. yd. for openings? *Ans.* 19.44\frac{4}{9}$.

14. Find the cost of plastering the walls and ceiling of a room 15 ft. long, 12 ft. wide, and 9 ft. high, at 15 cents a sq. yd., allowing 6 sq. yd. for openings. *Ans.* $10.20.

15. The signal service reports that 3$\frac{1}{2}$ in. of rain fell in 24 hr.; how many gal. of water fell on an acre of ground?
Ans. 95040.

16. What is the cost of papering the walls of a hall 36 ft. long, 24 ft. wide, and 18 ft. high, with paper 1$\frac{1}{2}$ ft. wide, at $2.50 a roll of 16 yards, allowing 64 sq. yd. for doors and windows? *Ans.* $55.

17. A roll of wall paper 8 yd. long and 18 in. wide costs 25 cents. What will be the cost of paper for the four walls of a room 30 ft. by 27 ft. by 9 ft., no allowance being made for openings? *Ans.* $7.12½.

18. Find the cost of plastering the walls and ceiling of a room 14 ft. long, 10 ft. wide, and 9 ft. high, at 35 cents a sq. yd., making an allowance of one-half for 2 doors, each 3 ft. by 7 ft., and 3 windows, each 2 ft. 8 in. by 6 ft. *Ans.* $20.80.

19. A man buys 2 acres of land at $600 an acre; he sells 15 lots, 15 ft. by 100 ft., each at $150 a lot, and the remainder of the land at 5 cents a sq. ft. Find his entire gain. *Ans.* $4281.

20. How many cubic feet of stone are required to build a wall 2 ft. thick and 6 ft. high about a rectangular cellar whose interior dimensions are 20 ft. by 16 ft.? *Ans.* 1056 cu. ft.

21. Find the time required to fill a cistern 8 ft. square and 5 ft. deep by a pipe which admits water at the rate of 1 quart a second. *Ans.* 2 hr. 39 min. 35 sec.

22. A certain walk is 100 ft. long and 5 ft. wide; the walk is of 2-inch hemlock plank, laid crosswise on three stringers, each of which is 2 in. by 4 in. Find the amount of lumber in the walk and its cost, at $15 per 1000 ft. *Ans.* 1200 b. ft., $18.

V.

1. Bought hats at $27 a dozen and sold them at $3.75 each; find the gain per cent. *Ans.* 66⅔%.

2. Hats bought at $15 a dozen are sold at $2 apiece; find the gain per cent. *Ans.* 60%.

3. If 10% is lost by selling cloth at 28 cents a yd., for what must it be sold to gain 20%? *Ans.* .37⅓.

4. By selling a horse for $144 a profit of 60% is made; find the cost of the horse. *Ans.* $90.

5. If cranberries are bought at $4 a bushel, at what price per quart must they be sold in order to gain 20%? *Ans.* .15 a quart.

6. Bought 1864 lb. of hay at $12 a ton and sold it at 80 cents a hundredweight; find the whole gain and the gain per cent. *Ans.* 33⅓%.

7. A house was sold for $7050, at a loss of 6%; for what price should it have been sold in order to gain 15%? *Ans.* $8625.

8. Bought 86340 lb. of coal at the rate of $3.60 a gross ton and retailed it at 42 cents a hundred pounds; find the gain per cent. *Ans.* 161%.

9. A man pays $75 for insuring his house for ¾ its value at 1¼%; find the value of the house. *Ans.* $8000.

10. An agent receives $541.62 with which to buy flour; how much will he buy, at $4.50 a barrel, after deducting his commission of 2%? *Ans.* 118 barrels.

11. A house and lot cost $5000; the insurance is $25, taxes are $50, and repairs $75 annually; what rent must be received in order to realize 6% on the investment? *Ans.* $450.

12. A house worth $4500 is insured for ⅔ of its value at the rate of ¼ of 1% per annum; repairs cost $50 and taxes $42.50 a year. What must be the rent per month to realize 6% on the investment? *Ans.* $30⅚.

13. I buy goods at commercial discount of 15 and 10 from the list price and sell at the list price; find the per cent. profit. *Ans.* 30⅓%.

14. A merchant sold a case of goods which cost $14.40 at 10% below the marked price, thus gaining 25% on the cost; find the marked price. *Ans.* $20.

8.

1. At what price must 5% bonds be bought so as to realize $7\frac{1}{2}$% on the investment? *Ans.* .$66\frac{2}{3}$.

2. At what price must 4% stock be bought so that the investment may yield 5%? *Ans* 80%.

3. Find the rate of income on 4% bonds bought at 115. *Ans.* .$03\frac{11}{23}$.

4. What is the per cent. profit on stock bought at 90 and sold at 110? *Ans.* $22\frac{2}{9}$%.

5. Find the rate per cent. on an investment in government 3% bonds bought at 115. *Ans.* .$02\frac{14}{23}$.

6. What is the rate per cent. of income on an investment in 4% bonds bought at 110? *Ans.* .$03\frac{7}{11}$.

7. By selling stock at 84 there is a gain of 5% on the investment. At what price was the stock purchased? *Ans.* 80.

8. At what price should $4\frac{1}{2}$% bonds be bought to make the income from investment equivalent to that from 3% bonds at par? *Ans.* 150.

9. What per cent. would I receive on my investment if I should buy at 10% discount stock which pays an annual dividend of $4\frac{1}{2}$%? *Ans.* .05.

10. I buy stocks at 80 and sell them at par. Find the per cent. profit. *Ans.* 25%.

11. I buy stocks at 4% discount and sell at 4% premium. What per cent. profit do I make on the investment? *Ans.* $8\frac{1}{3}$%.

12. If 10 shares of stock paying 8% are sold at 175, and the proceeds loaned at 5%, will the income be increased or diminished, and how much? *Ans.* 7.50.

13. If 4% bonds to the amount of $8000 face value are bought at $92\frac{1}{8}$%, find the cost of the bonds and the rate of income on the investment. *Ans.* \$7400, $4\frac{10}{37}$%.

14. A capitalist buys U. S. 4% bonds to the amount of \$50000 par value at $112\frac{3}{8}$, brokerage $\frac{1}{8}$%. Find the cost of the bonds and the rate of income on the investment. *Ans.* $3\frac{5}{9}$, \$562550.

15. A speculator buys bonds whose par value is \$10000 at $113\frac{3}{8}$ and sells them at $115\frac{1}{8}$. How much does he gain if brokerage is $\frac{1}{8}$% in each transaction? *Ans.* \$150.

16. Find the number of shares of stock at $30\frac{3}{8}$% discount that can be bought for \$2650.50, brokerage $\frac{1}{8}$%. *Ans.* 38 shares.

17. United States 4% bonds, to the amount of \$20000 face value, are bought at $122\frac{5}{8}$, brokerage $\frac{1}{8}$%. Find the cost of the bonds and the rate of income on the investment. *Ans.* $3\frac{127}{491}$, \$24550.

9.

Find the amount of:

1. \$864 for 1 yr. 3 mo. 15 da. at 5%. *Ans.* \$919.80.

2. \$375 for 11 mo. 17 da. at $4\frac{1}{2}$%. *Ans.* \$391.27.

3. \$486.50 for 1 yr. 5 mo. 17 da. at $5\frac{1}{2}$%. *Ans.* \$552.52.

4. \$436 from January 1, 1893, to June 13, 1893, at $4\frac{1}{2}$%? *Ans.* \$444.829.

5. \$1750 from September 23, 1897, to June 14, 1898, at 5%. *Ans.* \$1813.44.

6. \$4356:30 from January 1, 1899, to June 15, 1899, at $5\frac{1}{2}$%. *Ans.* \$4465.96.

7. \$1500 from January 21, 1898, to June 10, 1898, at $5\frac{1}{2}$%. *Ans.* \$1527.19.

8. A sum of money at 5% simple interest amounts in 1 yr. 9 mo. and 18 da. to $1395.20; find the principal.
Ans. $1280.

9. A sum of money at 4½% simple interest amounts in 1 yr. 7 mo. and 18 da. to $375.725; find the principal.
Ans. $350.

10. What is the true discount and the present worth of $412, due in 6 mo. without interest?
Ans. $12, $400.

11. A bank note for $150 at 90 da. is discounted at a bank on the day when it is made; find the proceeds.
Ans. $147.67.

12. What are the proceeds of a note for $900, dated December 1, payable in 60 da., and discounted December 15, at 6%? *Ans.* $892.65.

13. Find the proceeds of a note for $350, without interest, dated May 1, 1897, payable in 4 mo. and discounted July 16 at 5%. *Ans.* $347.57.

14. The proceeds of a note, due in 4 mo. and discounted at a bank at 6%, are $450.80; what is the face value of the note? *Ans.* $460.30.

15. Find the face of a bank note at 90 days that the proceeds may be $300. *Ans.* $304.70.

16. The proceeds of a note discounted at a bank for 90 days at 6% are $500; find the face of the note.
Ans. $507.77.

17. $300 was loaned September 1, 1897; on January 1, 1898, a payment of $100 was made; find the amount due to March 25, 1898. *Ans.* $208.41.

18. On a note for $400, at 6%, dated January 12, 1899, the following payments have been made: May 22, 1899, $200; October 2, 1899, $150. Find the amount due March 30, 1900. *Ans.* $6505.

10.

1. If 7 bushels of wheat cost $6.30, what will 11 bushels cost? *Ans.* $9.90.

2. If 3 horses eat 6 bu. of oats in 8 da., how many bushels will 5 horses eat in 12 days? *Ans.* 15 bushels.

3. If 18 men can dig 128 yd. of ditch in 32 da., how many yards can 12 men dig in 64 days?

Ans. 170⅔ yards.

4. If a stick of timber 6 inches square and 20 ft. long costs $1.50, what will be the cost of a stick 8 in. square and 30 ft. long? *Ans.* $4.

5. If 27 men working 10 hr. a day can build a wall in 14 days, how many hours a day must 12 men work to build the wall in 45 days? *Ans.* 7 hours.

6. If 3 men can do a certain piece of work in 6 da. of 10 hr. each, in how many days of 8 hr. each will 5 men do the same work? *Ans.* 4½ days.

7. If a man traveling 9 hr. a day goes 405 miles in 15 days, how far can he go in 25 da., traveling 8 hr. a day at the same rate per hour? *Ans.* 600 miles.

8. If it costs $80 to plow a field 40 rd. by 80 rd., when we pay $5 a day for a man and team, how much will it cost to plow a field 30 rd. by 60 rd., if we pay $4 a day?

Ans. $36.

9. If 3 men can dig a trench 10 rd. long, 2 ft. wide, and 5 ft. deep in 4 da.; how many men will it take to dig a trench 40 rd. long, 2 ft. wide, and 5 ft. deep in 8 da.?

Ans. 6 men.

10. If 3 men in 12 days of 10 hr. each can build a wall 100 ft. long, 14 ft. high, and 3 ft. thick, how long will it take 4 men working 8 hr. a day to build a wall 200 ft. long, 16 ft. high, and 4 ft. thick? *Ans.* 34$\frac{2}{7}$ da.

11.

1. A field containing 25 acres of land is 4 decimeters wide; find its length. *Ans.* 252.926 + km.

2. How much iron weighing 48 kilograms to the meter will be required for a double-track railway 150 kilometers in length? *Ans.* 14400000 kilo.

3. If rain falls to a depth of 15 millimeters on a level, how many kilograms of water fell on a square kilometer of ground? *Ans.* 15000000 kg.

4. Find the contents in hectoliters of a bin 7.6 meters long, 4.3 decimeters wide, and 386 centimeters deep. *Ans.* 126.14480 hectoliters.

5. Find in liters the capacity of a cistern 25 decimeters square and 3 meters deep. *Ans.* 18750 liters.

6. Find in kilograms the weight of the water that fills a cylindric tank 1 meter high and 60 centimeters in diameter. *Ans.* 282.857 kg.

7. Find the weight in grams of a bar of iron 3.2 meters long, 1.5 decimeters wide, and 2 centimeters thick (weight of 1 cu. dec. of iron = 7.8 kilograms). *Ans.* 74880 grams.

8. What is the weight in kilograms of the water which fills a tank whose interior is 380 centimeters long, 23⅘ decimeters wide, and .35 meters deep? *Ans.* 3165.40 kg.

9. Find the cost of paving a walk 140 centimeters wide and ⅗ kilometers long, at $1.25 a square meter. *Ans.* $1050.

10. A tank is 6.3 meters long, 35 decimeters wide and 420 centimeters deep; if a stream of water flows into the tank at the rate of 3.6 liters a minute, in how many minutes will it be filled? *Ans.* 25725.

11. Find in kilograms the weight of a bar of lead 72 decimeters long, 4½ centimeters wide and 25 millimeters thick, lead being 11.35 times as heavy as water.

Ans. 91.985 kg.

12. A railway train runs 500 meters in 20 seconds; how many kilometers will it run in an hour? How long will it take to run 150 kilometers? *Ans.* 90 km., 1⅔ hr.

13. A rectangular cistern is 4 meters long, 3 meters wide, 2½ meters deep; find its capacity in hectoliters and the weight in kilograms of the water which it will hold.

Ans. 300 hl., 30000 kg

14. A room which is 6.5 meters long by 4.5 meters wide is to be covered with carpet 7.5 decimeters wide; how many meters in length of carpet will be required?